Mathematical Considerations		
Old Habit to be eliminated	New Habit to be adopted	Why?
Defining Equality as "Same as" Ex: 'remember students the equals sign means same as'	Defining Equality as "Same Value As" Ex: 'remember students the equals sign means same value as. The two values do not have to look alike, but they will have the same value. 3 + 4 tells a different math story than 4 + 3, but we know that they will both yield the same value of 7, so they are equal. Are they exactly the same? No! But they are equal.'	The definition 'same as' is mathematically incorrect and leads to misconceptions and blurry number sense. Equals means that two things are the same based on one attribute—their quantitative value. Just as red or rough is an attribute, so is something's quantitative value an attribute. It is as absurd to say that 4 + 3 is the same as 1 + 6 as it is to say that a red truck is the same as a red lollipop. In the former they have the same value, in the latter they have the same color. In neither instance are they 'the same thing' as each other. Be precise.
Calling digits 'numbers' Ex: 'yes, the numbers 7 and 3 are in 73.'	Call digits digits and call numerals numerals. **Digits** are in numerals. **Numerals** are a written symbol that represent **Numbers.** Ex: 'no, remember, that 7 is a digit that represents, in this instance, 7x10 or the number 70, it is not the 'number' 7.' 73 has two digits, 7 and 3. Use the word 'number' when you are referring to the entire value that a digit represents and use the term digit when you are referring simply to the digit without regard to its exact value within the numeral. 73 is a numeral that represents the number value of 73. 7 is a digit, 3 is a digit within that numeral. In standard form, 73 is composed of the numbers 70 and 3. We get those numbers by multiplying our digits by a power of ten, depending on where they are located. 7x10 yields 70, for instance.	It is MOST important to make the distinction between Digits and Numbers/Numerals. Numbers and Numerals are in essence the same with Numerals being the written form of the idea of the number. On the other hand, Digits and Numerals are not, in essence, the same. When we are *imprecise* about digits and numerals we *lose* an opportunity to reinforce the workings of the base-ten number system. 7 is not a number when it is contained in 73. It does not mean '7' in the number sense. It means 7 in the digit sense and must be multiplied by a power of ten to be fully understood. *Being clear here with students opens*

		up these base-ten conversations.
Addition 'makes things get bigger'	'We use addition when we are combing two or more parts to make a total or when we are comparing pieces of information to figure out a total.'' OR When we add we are taking a decomposed number and composing it into a simplified form.	Addition does not 'make things get bigger'. Addition is about combining quantities and it is only in elementary school where the numbers we combine are all positive numbers. By saying that addition makes things get bigger we are 1) saying something that will have to be 'debugged' in middle school 2) covering up an opportunity to discuss the actual structures of addition. If I combine the 8 dollars I have with the 2 dollars I owe you, I will end up with 6 dollars. This is not more than my initial 8.
Subtraction 'makes things get smaller'	'We use Subtraction when we are finding a missing part of a total or when we compare two numbers to find the difference between the two.'	Subtraction does not 'make things get smaller'. As above, this is a false construction based on a limited set of numbers that are introduced in elementary school. For instance: 5 - - 3 = 8. Consider in your own life the situation where a debt is taken away from you: "don't worry about that three dollars you owe me…". Also consider the comparative model. If I compare what I have, let's say my net worth is $10,000, to what you have, let's say your net worth is, uh oh… $- 5,000. When I compare those two numbers, the difference between our net worth is $15,000. This is actually the greatest of those three numbers!
'We don't have enough ones so we need to go to the next place'	'In this form, we don't have access to the ones we need so we need to change the form of the number' or 'we don't have access to the ones we need and we need to get at those ones' or 'we have plenty of ones,	Consider the research by Constance Kamii that indicates that elementary students in the United States do not understand that there are ten ones in a ten. I believe that the cultural habit we have of saying 'we don't have enough ones' trumps our efforts to show students with manipulatives that there are ones inside tens. **Language matters!** Students

	look 25 is a larger number than 18 so we will even have some left over, but we need to decompose the higher unit value here so we can get at some of the ones that are composed into tens.'	are literal. If they hear you say 'we don't have enough ones' whenever these problems are addressed, they begin to believe you! Shift your language to habitually point out that there are ones available within the number, but that the form of the number is the issue. The ones need to be accessed by decomposing the higher unit value.
'You can't take a big number from a little number'	'We could take a larger number from a smaller number, but we would get a negative number… you will learn about these later, but right now we will learn to solve this problem using all positive numbers.'	This problem language is the cousin of the 'we don't have enough ones' mistake. It is critical that we do not say things that are mathematically inaccurate in the service of a procedure or algorithm. We can both teach efficient algorithms AND maintain mathematically accurate language. Kids respond to this much better than we expect. At the very least, they hear accurate mathematical language from you, and, often times, this precision on your part leads to great conversations with the class. Here note the following: 25 -18 -3 10 7 All subtraction problems of the type taught in second grade could actually be solved by taking a larger number from a smaller number!
'Let's 'borrow' from the tens place'	'Here we will need to 'decompose a higher unit value' to get the number into the form we want…'	This is the crème de la crème of subtraction with re-grouping language because it prepares students both for situations beyond tens and ones (we decompose the higher unit 1,000 into ten 100s for instance) AND for situations with fractions and beyond (for instance, I decompose the higher unit value here, when I need to change the form of this mixed fraction to gain access to more thirds to solve this problem: 3 4/3

		~~4~~ 1/3 - 1 2/3 When I decompose the higher unit (ones) into the lower unit (thirds) I am acting on the exact same mathematical structure as with tens and ones. Our language should connect those ideas so that it does not appear to be a novel concept when taught in later grades.
'Re-group/Trade'	Compose the lower unit value/ Decompose the higher unit value.	In these cases, the most important change is that you eradicate the misleading 'borrow' from your vocabulary. The terms re-group and trade are often times very communicative and can match the manipulative you are using. Do the best you can, however, to also include the concept of decomposing and composing the higher unit value as well as this language. The idea of decomposing a higher unit value and composing a lower unit value taps into a common 'move' in mathematics. For instance, when I want the number 1 ¼ in improper form I must decompose the higher unit value (1) in a lower unit value (4/4). Any time you can emphasize the unit, you should try and do so.
Multiplication 'makes things get bigger'	"There are three main structures for multiplication. One is that multiplication is repeated addition (measurement model), the second answers the question how many different unique possibilities do have when matching one set with another (Fundamental Counting Principle) and the third involves finding a total amount or total area when a column and row or two sides are known (Area Model).' and then perhaps… 'Create a story problem for each	Multiplication only 'makes things get smaller' in the limited world of positive whole numbers. As above, focusing on this idea that an operation 'does something' to something else distracts from conversations about the structure of the expression and the equality of the number sentence. Consider 4 x ½ or 4 * -1.

	different structure type with the expression 4 x 3…'	In both cases the product has a lesser value than the first factor.
Division 'makes things get smaller'	"There are three main structures for division. One structure involves repeated subtraction of groups (measurement model), the second answers the question "how many for each one" (partitive model/unit rate model) and the third model for division (area model) involves finding a side when an area and another side are known. Here, we are focusing on (name the structure) type of division…"	Division has just as much chance of 'making things smaller' as it does of 'making things larger.' In early grades, with only whole numbers to consider, it looks as if division behaves this way, or has that power. But this limited sample makes us think a behavior exists that does not exist! Your *divisor,* in combination with the act of dividing, determines the relative size of the quotient, not the operation of division itself. Consider: $6 \div 1/2 = 12$. Or $-6 \div -2 = 3$ In both cases here, the quotient is a larger number than the dividend.
7 (for example) 'doesn't go into 3' (for example).	'We can divide 3 into 7 but the result will not be a whole number. When you begin working with fractions you will solve problems like this regularly. Here we want to consider numbers that divide into other numbers without creating fractional parts or 'left over pieces'.' And, in explanation… 'Well consider if we had 7 cookies and needed to split them between 3 people, what would happen? What about if we had 9 cookies between 3 people… In both cases we CAN split up the cookies, but which one was easier? Why?'	Again, we need to make sure we maintain our accurate mathematical language even when something will always be true at our grade level. It just is not true that 7 'doesn't go into' 3. Even young children can understand the idea that in some cases there is a 'cookie left over' that needs to be cut up in order for everyone to have equal shares. They know that I can have 7 cookies and split them between 3 people intuitively. The language that says 'you can't do that' separates their intuitive understanding with their academic mathematical understanding and our job is to **connect** the intuitive with the academic.
" 'and' means decimal point"	'When listening to and naming a number, consider the unit size(s) that are being communicated. As you know, 'and' generally means to combine things. When people call numerals 'and' means to combine, but it also triggers/communicates a change in unit size. This is particularly important to	And doesn't mean 'decimal point'. This is an artificial construction. In common parlance and in math parlance, and generally means to combine, to add to, to augment. There is no reason to limit people's way of naming numbers or reading

	think about when verbally communicating numbers with a decimal point in it.' Ex: "one hundred AND forty nine" means literally 1 hundred unit and 49 ones units (remember, if we don't name a unit our 'default' unit is ones)" "one hundred forty nine" means 149 ones. Say what you mean. But if the form of the number does not matter to the case or argument, then it is fine to call a number in any form. In this same way, I could call 1,500 as 'fifteen hundred' or 'one thousand five hundred'. They imply different forms (literally 15 hundreds units or 1,500 ones units), but they name the same number. If a decimal number has a fractional part, you need to say something to the listener to indicate a combination of ones units (or above) and some fractional units. "And" does this nicely: one hundred **and** forty nine hundredths. Here the speaker is saying 100 (implied ones unit) AND (shift units) 49 100ths units. 'fifteen hundred and forty nine hundredths' communicates 15 hundred units and 49 hundredth units. 'One thousand and five hundred and forty nine hundredths' is clunky, but it communicates the correct number with correct shifts in units from 1 thousand unit to 5 hundreds units to 49 hundredth units. And does not mean 'decimal point' but it does communicate to the listener a change in unit size when in the context of naming numbers or reading numerals.	numerals. The key is that the numeral reader or number-namer comprehends the number they are communicating and does so in a fashion that allows the listener to comprehend the value as well. If I say one hundred and forty five, you know what I mean. I have effectively communicated the value 145 because and means, quite clearly, a combination of my hundred and my forty-five ones. When I say 'one hundred and forty five and thirty seven one hundredths' I effectively, communicate the number I mean: 145.37. It may be a matter of taste to say that 'one hundred forty five and thirty seven hundredths' is the most elegant way to say this number, but that is different than claiming that it is the 'only' or 'best' way to name it. To construct the idea that 'and' 'means' decimal point is inadvisable for two reasons: 1) it is not correct from a language perspective and 2) it buries the opportunity to have a discussion that focuses on considering unit size. Whenever you get the chance to talk about units, you want to do so!
"Cancels out" Ex: 'These 8s 'cancel out''	Explicitly use/discuss the property or idea that allows you to simplify. Ex: "Here I have an 8 divided by an 8 and we know that anything divided by itself equals 1. So if I have 1 times something what property can I use? ... Yes, the Multiplicative Identity Property. So you can see here that we have simplified this expression without disturbing/changing its value... "Here I have an X being subtracted from an X and I know that anything minus itself equals 0. So, here I have 0 + something else. What property can I use here to make this expression simpler without changing its value? ... Yes, the	When we bury these properties under the term 'cancels out' students do not notice how often the properties are used and how important they are. Asking students to memorize the properties because they are important, and then not pointing out the properties when they are used sends a mixed message to students that the properties are just facts rather than **tools** to be used regularly.

	Additive Identity Property.	
Pedagogical Considerations		
Old Habit to be eliminated	New Habit to be adopted	Why?
"The answer" Ex: What was the answer for the next question?	"The model" "the relationships" "the structure" "justify your answer" Ex: Who can show how they modeled the next question? Did someone else do it a different way? So what type of problem is this? Is it a part-part whole problem or a comparative problem? What in the story made you think that? …Is it one you would solve using the structure of a linear equation? How do you know that? …Is it a problem that involves a proportional relationship that you can use? What made you realize that? Is there another way to draw that relationship? Justify your answer for us and explain to us why you were able to 'switch' those numbers in that way…	When our classroom habit is to 'find answers' we forget to have the most important conversations of all: How did you do that? Why did you do that? What are the relationships that were important in this problem? How did you know that? What do mathematicians call it when you do that? Oftentimes math class becomes a series of seemingly unrelated puzzles to get an answer to rather than a place where important themes emerge. Discussing the relationships in the problem rather than the 'answer' helps to develop the important themes.
'Highlight the numbers and the Key Words'	'Lets read what is happening here and find the math main idea using our bar models/structures of equality'	This conversation is larger than can be addressed here. But key words are documented to be counter-productive. Consider adopting bar modeling as a habit of instruction.
Guess and Check as "a strategy" that you teach.	Systematic mathematical representations, such as the use of bar models, are what teach students to think precisely as mathematicians think. Using base-ten to make a problem simpler is also a mathematical strategy.	While guessing and checking sometimes involves engaging number sense to locate an answer, this is not an approach that should be taught as a 'strategy'. Mathematical strategies involve instruction that prepares students for more difficult mathematics where 'guessing and checking' becomes a deeply problematic approach.

		Again – We engage in this type of reasoning at times to access problems where we do not need precise answers. But our time with students should be spent developing strategies that 'scale up'.
To be developed:		
¼ read as '1 over 4'		
'Keep, Change, Flip'		
'Change Change'		
PEMDAS		
FOIL		