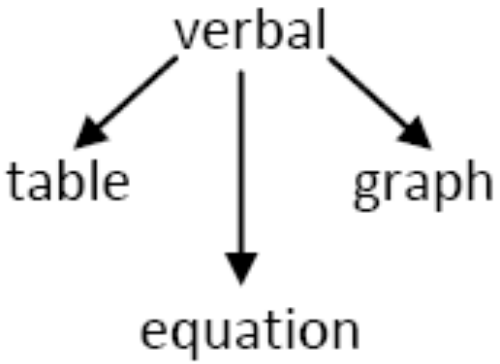
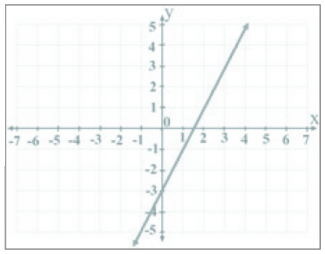
Algebraic Reasoning & Linear Equations

3rd Annual UMTSS
UTAH MULTI-TIERED SYSTEM OF SUPPORTS

June 25, 2015 Layton, Utah



x	y
–2	3
–1	1
0	–1
1	–3



$$y = mx + b$$

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North Carolina State
Teacher Education & Learning Sciences
Elementary Education
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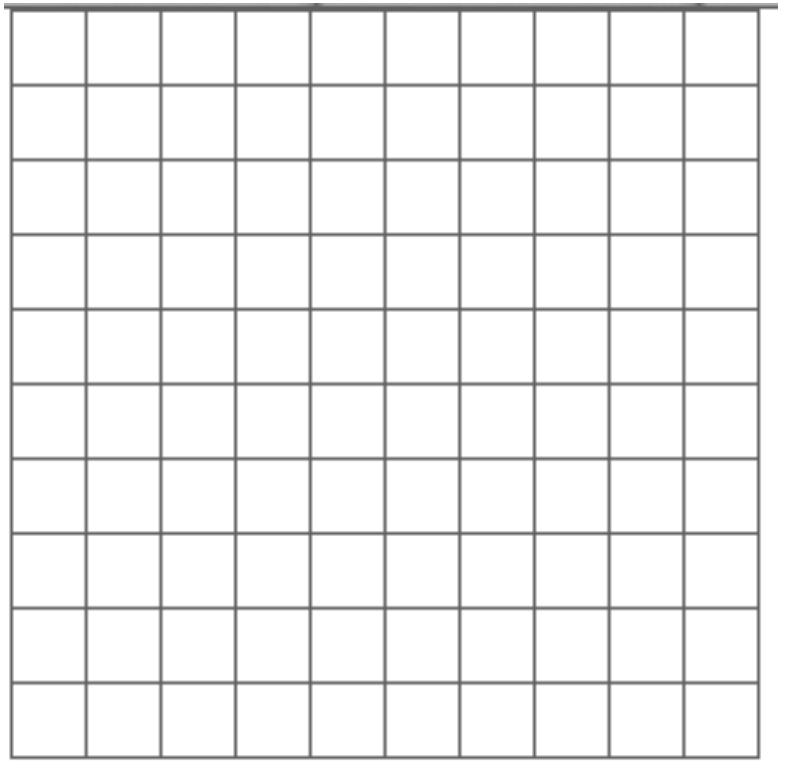
Agenda

1. Current condition
2. UDL and Core
3. The Brain and Math
4. **Conceptual Algebra in action**
5. Connections to advanced content
6. Reflections
7. How can I get this started in my o
8. Questions?

Solve
$3x + 4 = 10$

Graph
$3x + 4 = y$



How does your brain feel?

NAEP Task

Give the value of y when x = 3.

x	1	3	4	7	n
y	8		11	14	

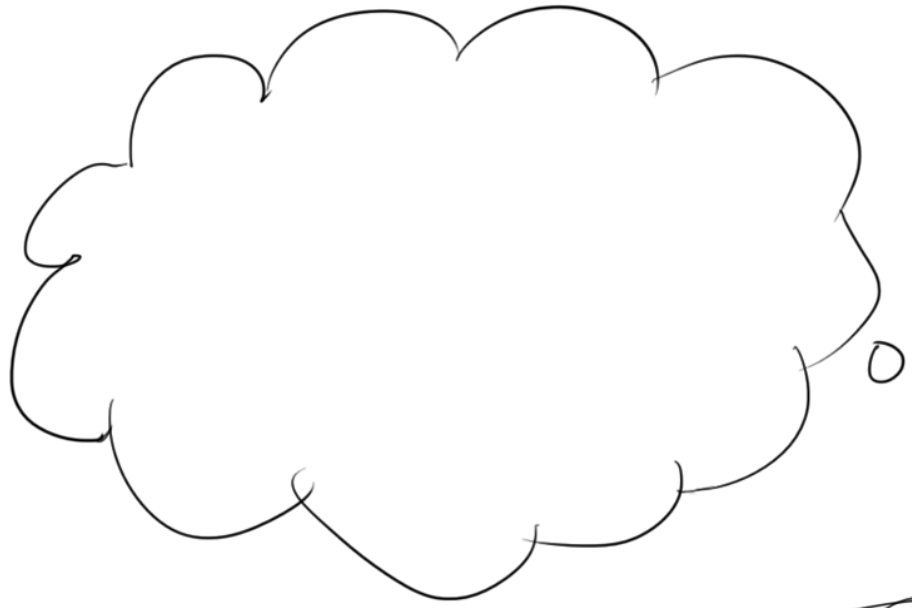
41% of students with one year of algebra could give generalized equation

3X +7 = Y

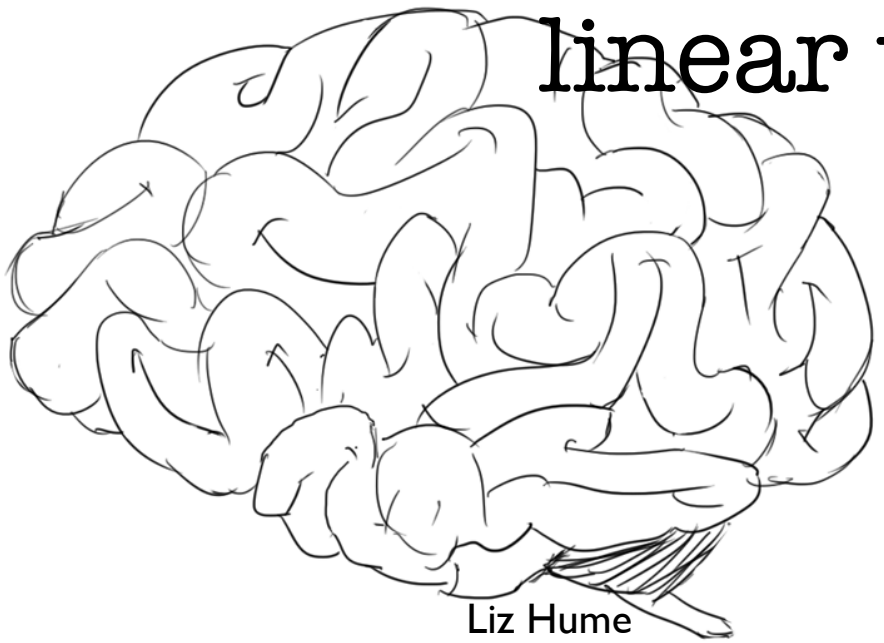
Write a story problem for this equation…

How would your students do?

7th grade?
8th grade?
Algebra 1?
Algebra 2?
Calculus?



What are we PICTURING when we grapple with issues that lend themselves to linear thinking?



Liz Hume

Two Themes

1. Central Conceptual Structure
2. Task Analysis: How does **our understanding** of Algebra affect how we **instruct**, what we **monitor,** what we **assess**?

CCSS-M: Algebra

Seeing **Structures** in Expressions

Creating Equations

Reasoning with Equations

Universal Design for Learning Guidelines

Provide Multiple Means of Engagement

Purposeful, motivated learners

Provide options for self-regulation
- Promote expectations and beliefs that optimize motivation
- Facilitate personal coping skills and strategies
- Develop self-assessment and reflection

Provide options for sustaining effort and persistence
- Heighten salience of goals and objectives
- Vary demands and resources to optimize challenge
- Foster collaboration and community
- Increase mastery-oriented feedback

Provide options for recruiting interest
- Optimize individual choice and autonomy
- Optimize relevance, value, and authenticity
- Minimize threats and distractions

Provide Multiple Means of Representation

Resourceful, knowledgeable learners

Provide options for comprehension
- Activate or supply background knowledge
- Highlight patterns, critical features, big ideas, and relationships
- Guide information processing, visualization, and manipulation
- Maximize transfer and generalization

Provide options for language, mathematical expressions, and symbols
- Clarify vocabulary and symbols
- Clarify syntax and structure
- Support decoding of text, mathematical notation, and symbols
- Promote understanding across languages
- Illustrate through multiple media

Provide options for perception
- Offer ways of customizing the display of information
- Offer alternatives for auditory information
- Offer alternatives for visual information

Provide Multiple Means of Action & Expression

Strategic, goal-directed learners

Provide options for executive functions
- Guide appropriate goal-setting
- Support planning and strategy development
- Enhance capacity for monitoring progress

Provide options for expression and communication
- Use multiple media for communication
- Use multiple tools for construction and composition
- Build fluencies with graduated levels of support for practice and performance

Provide options for physical action
- Vary the methods for response and navigation
- Optimize access to tools and assistive technologies

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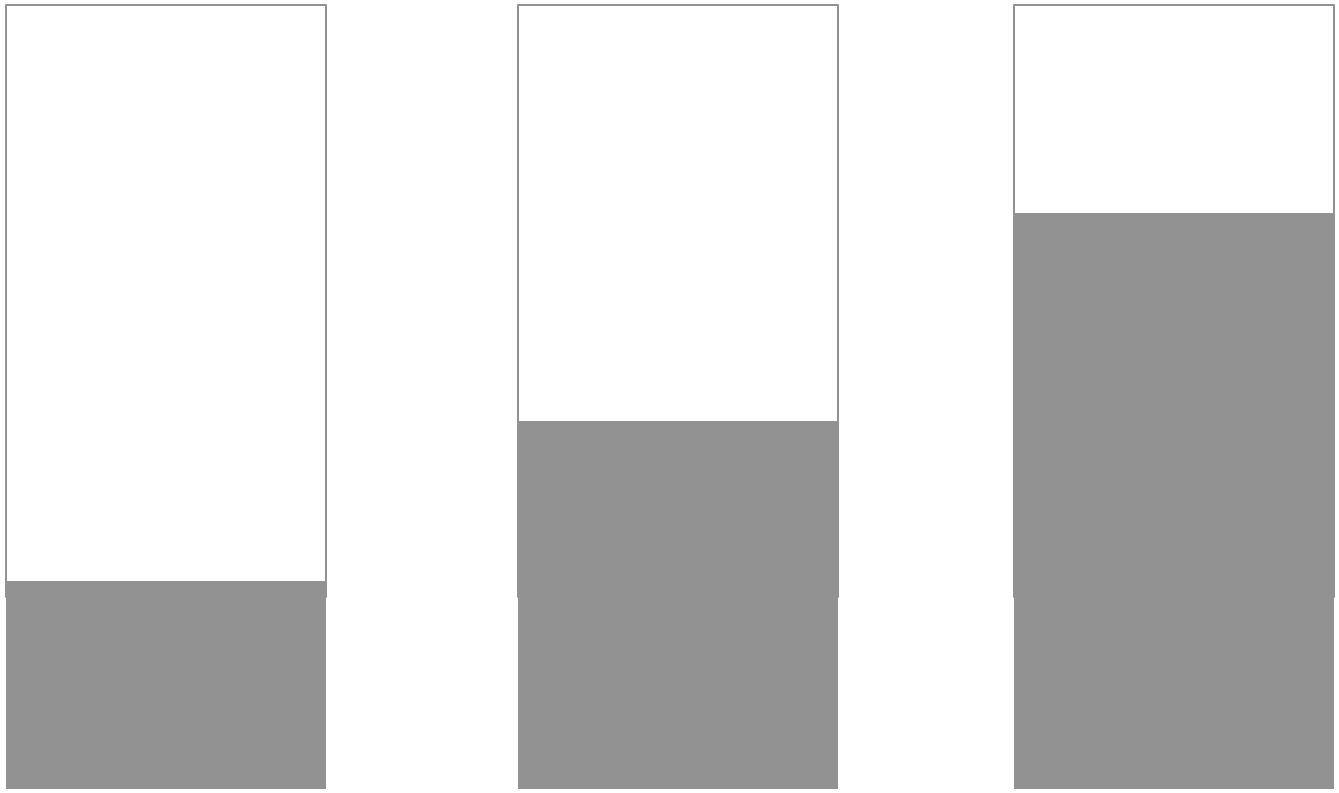
High Achieving Countries
MAKE CONNECTIONS

United States
TEACHES PROCEDURES

The Accumulator Model: Our Analog Brain



See Stanislas Dehaene 1999. The Number Sense

Gas Gauge picture From url:
http://www.marinepartssource.com/newdetails.asp?mfgno=57902P&pnumber=S57902P&mfg=TELEFLEX&desc=Amega%20Fuel%20Gauge

.78
Fuel Level

http://www.americanflyers.net/aviationlibrary/instrument_flying_handbook/chapter_4.htm

4.125 3.67 8.00

00.48 7.185 2.895

36,482 1400 9

32 1.75

In Order to Activate all parts of the Brain we must engage Magnitude and its connection to the Number Line...

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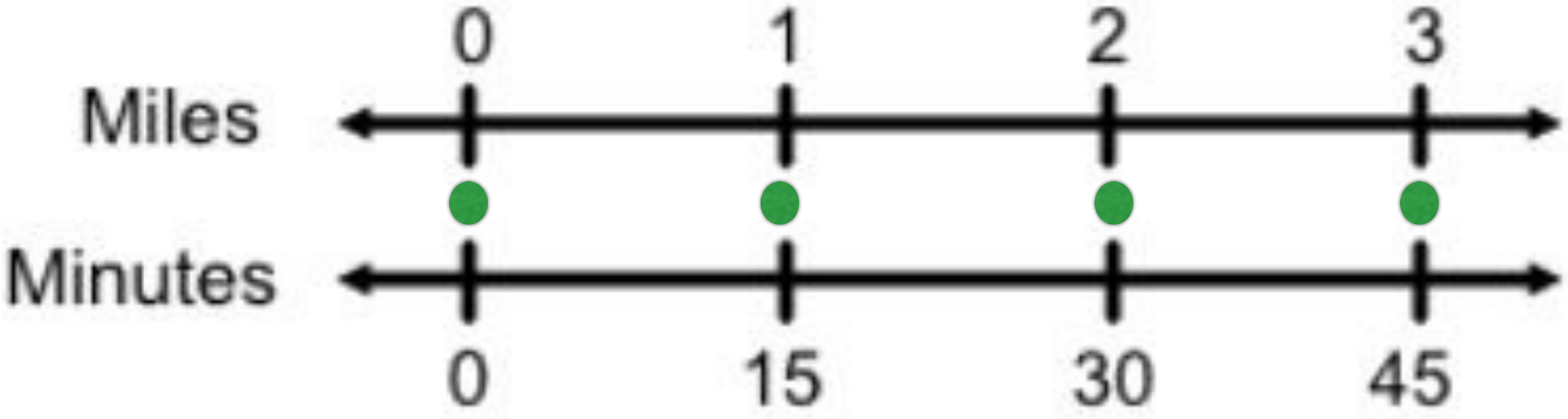
Central Conceptual Structure

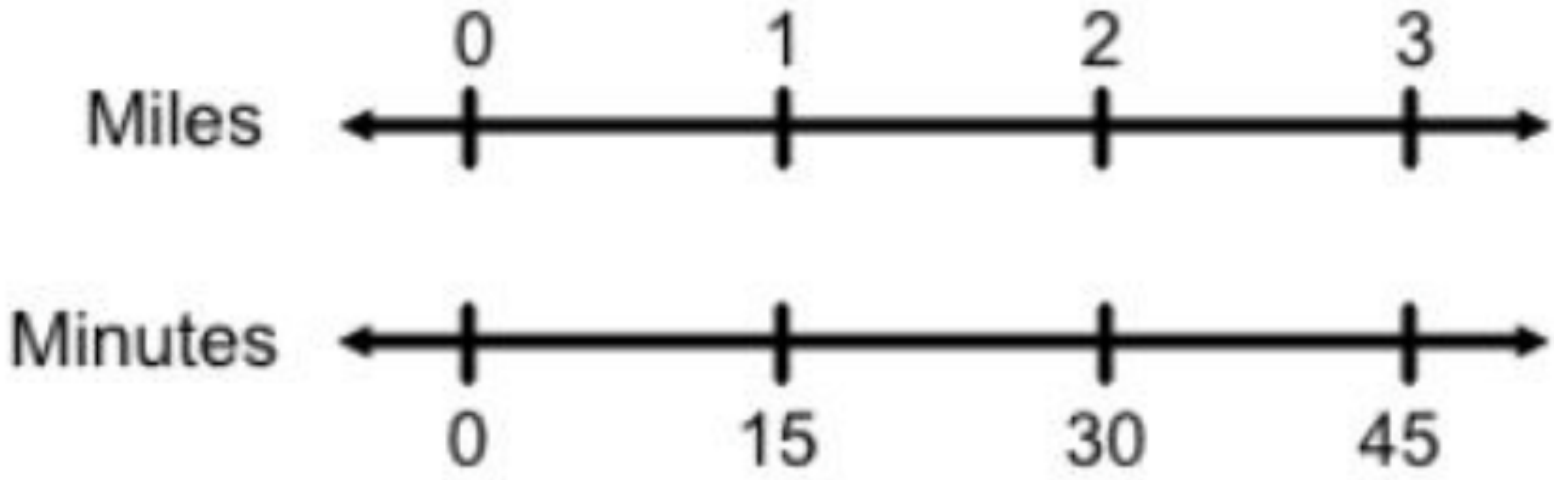
A Faulkiki Nerd Produ

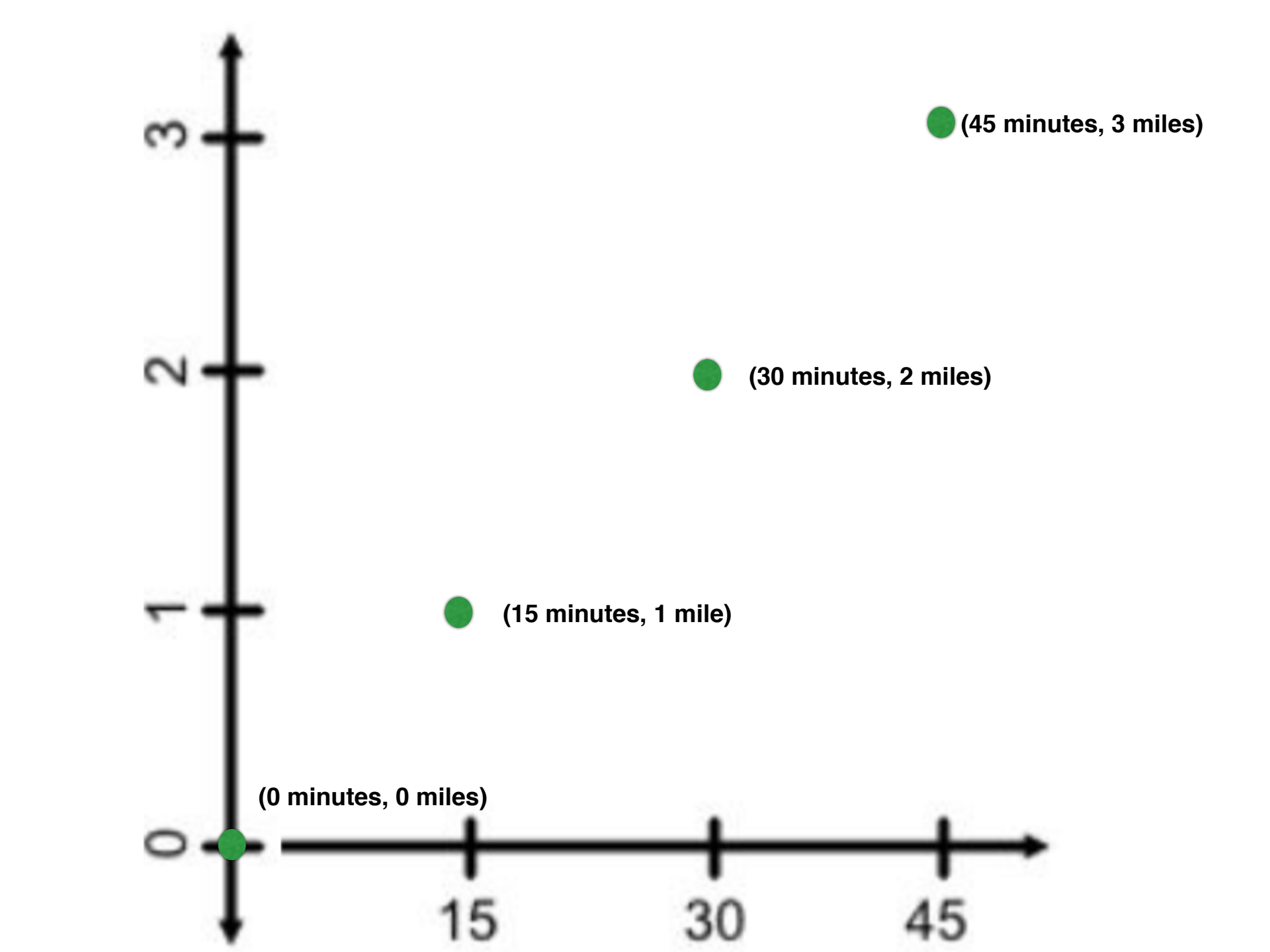
the **NATURE** of the instruction must change:

From Sharon Griffin
Number Worlds



http://quizlet.com


0
1
2
3
Miles
Minutes
0
15
30
45


3
2
1
0
15
30
45
(45 minutes, 3 miles)
(30 minutes, 2 miles)
(15 minutes, 1 mile)
(0 minutes, 0 miles)

The Sequence of Lessons you will see here were originally created by Valerie Faulkner while working for **Wake County Public School System in North Carolina.**

North Carolina Department of Instruction uses the sequence in the Math Foundations training co-written by Faulkner, Cain, Hale.

The lessons have been extended in **Walch Educational** Intro to High School Math and Foundations of Algebra both by Faulkner & Dupree.

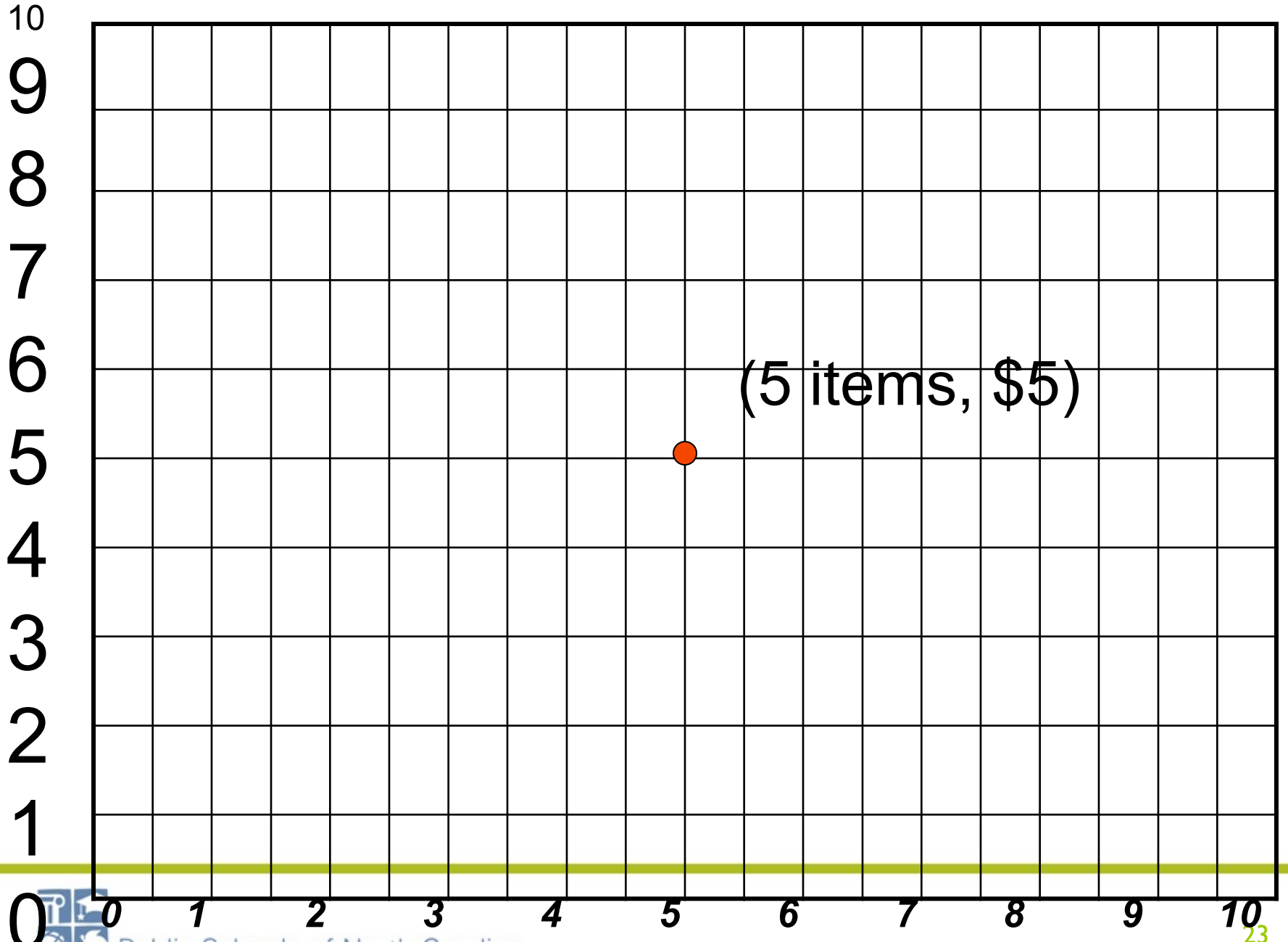
Dollar Deals

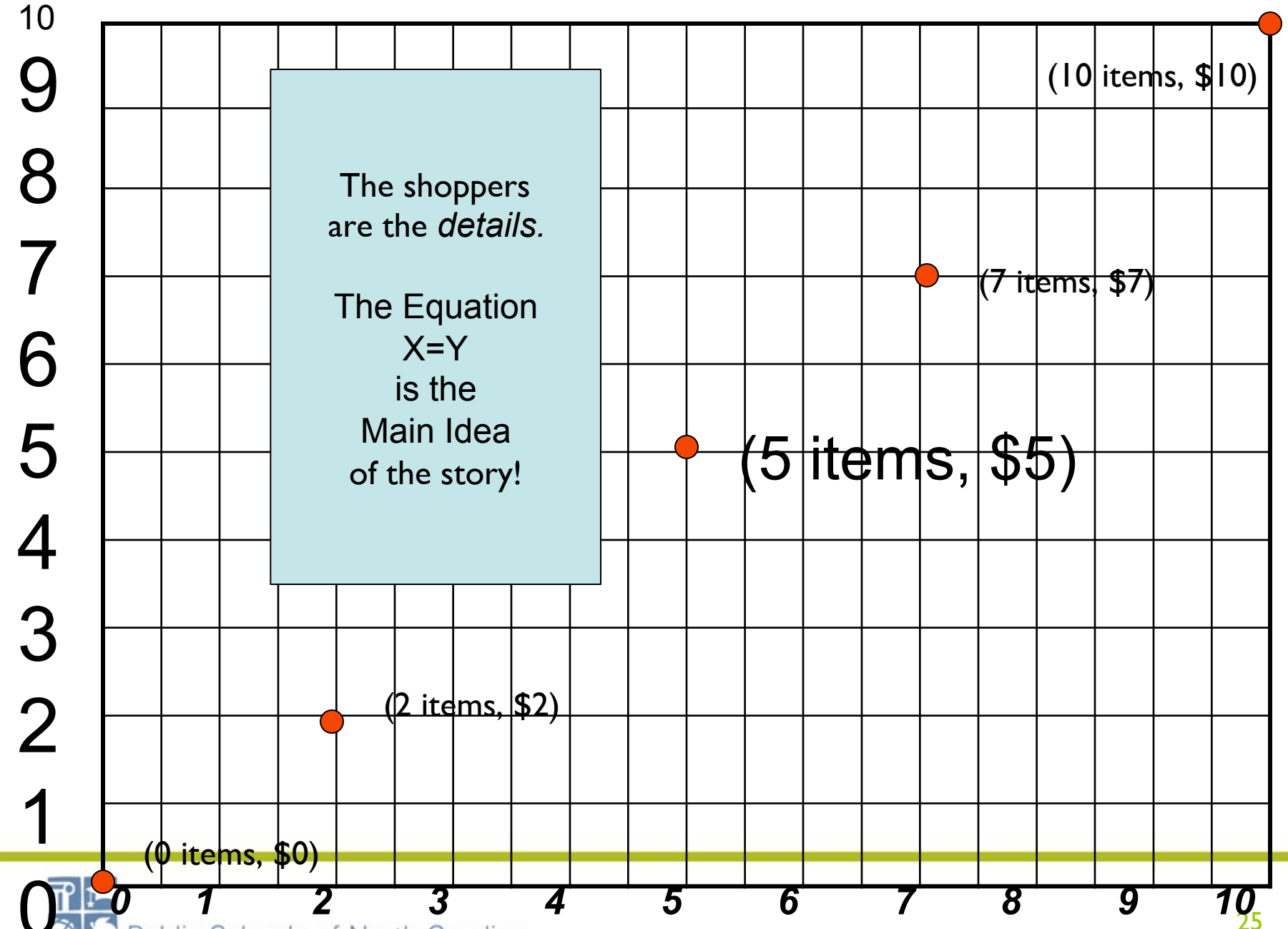
No Tax,

No Tricks:

One Dollar per One Item.

Where EVERYTHING is just one dollar.


10
9
8
7
6
5
4
3
2
1
0
(5 items, $5)
0
1
2
3
4
5
6
7
8
9
10


10
9
8
7
6
5
4
3
2
1
0
0
1
2
3
4
5
6
7
8
9
10
The shoppers are the *details.*
The Equation X=Y is the Main Idea of the story!
(10 items, $10)
(7 items, $7)
(5 items, $5)
(2 items, $2)
(0 items, $0)

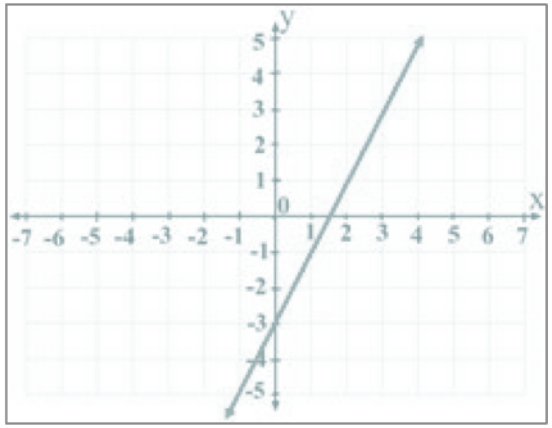
verbal

table

graph

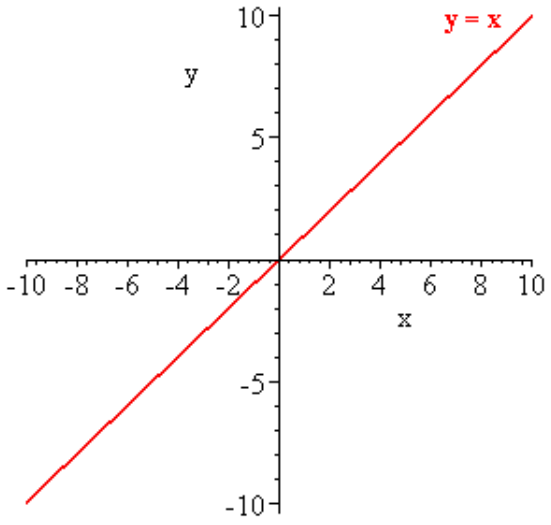
equation

x	y
–2	3
–1	1
0	–1
1	–3



$$y = mx + b$$

Identity Function



$$y = x$$

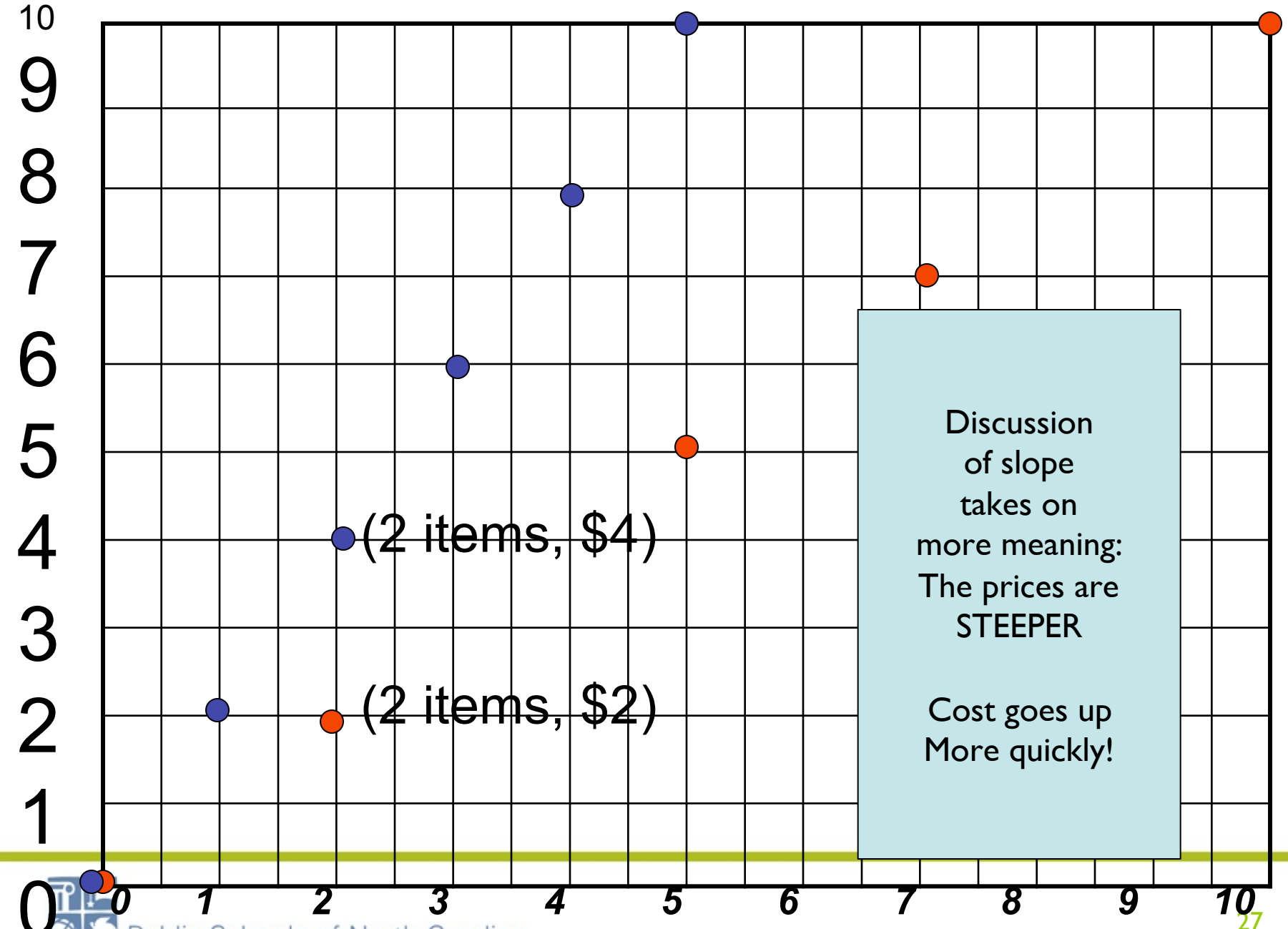
I went to the dollar store and bought 5 items so I know it will cost me $5

Puggly Wuggly Super Sale!

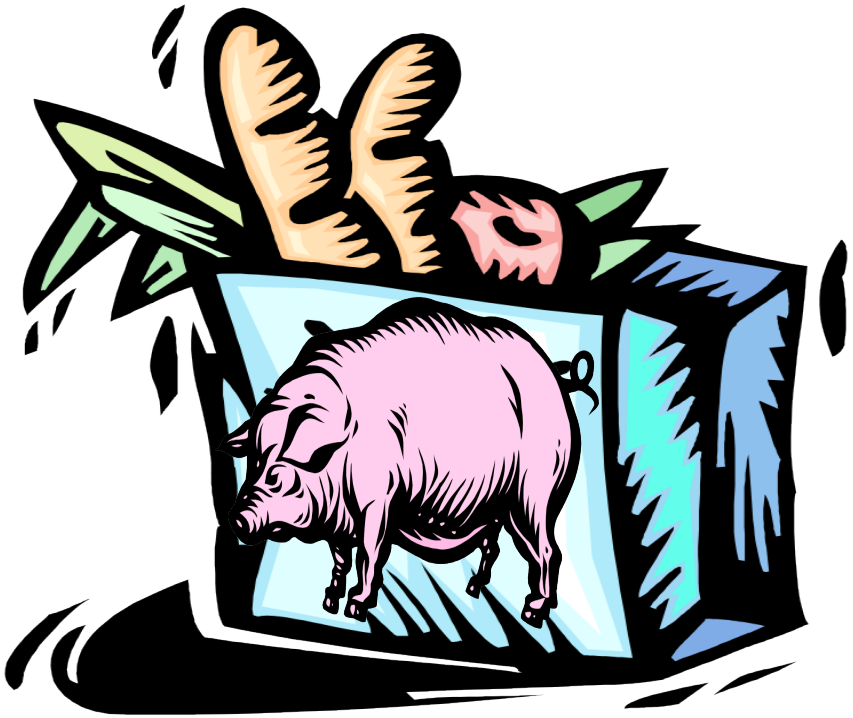
Bag day sale!

All items you can fit in 1 bag:

2 dollars per item!


10
9
8
7
6
5
4
3
2
1
0
0
1
2
3
4
5
6
7
8
9
10
(2 items, $4)
(2 items, $2)
Discussion of slope takes on more meaning: The prices are STEEPER
Cost goes up More quickly!

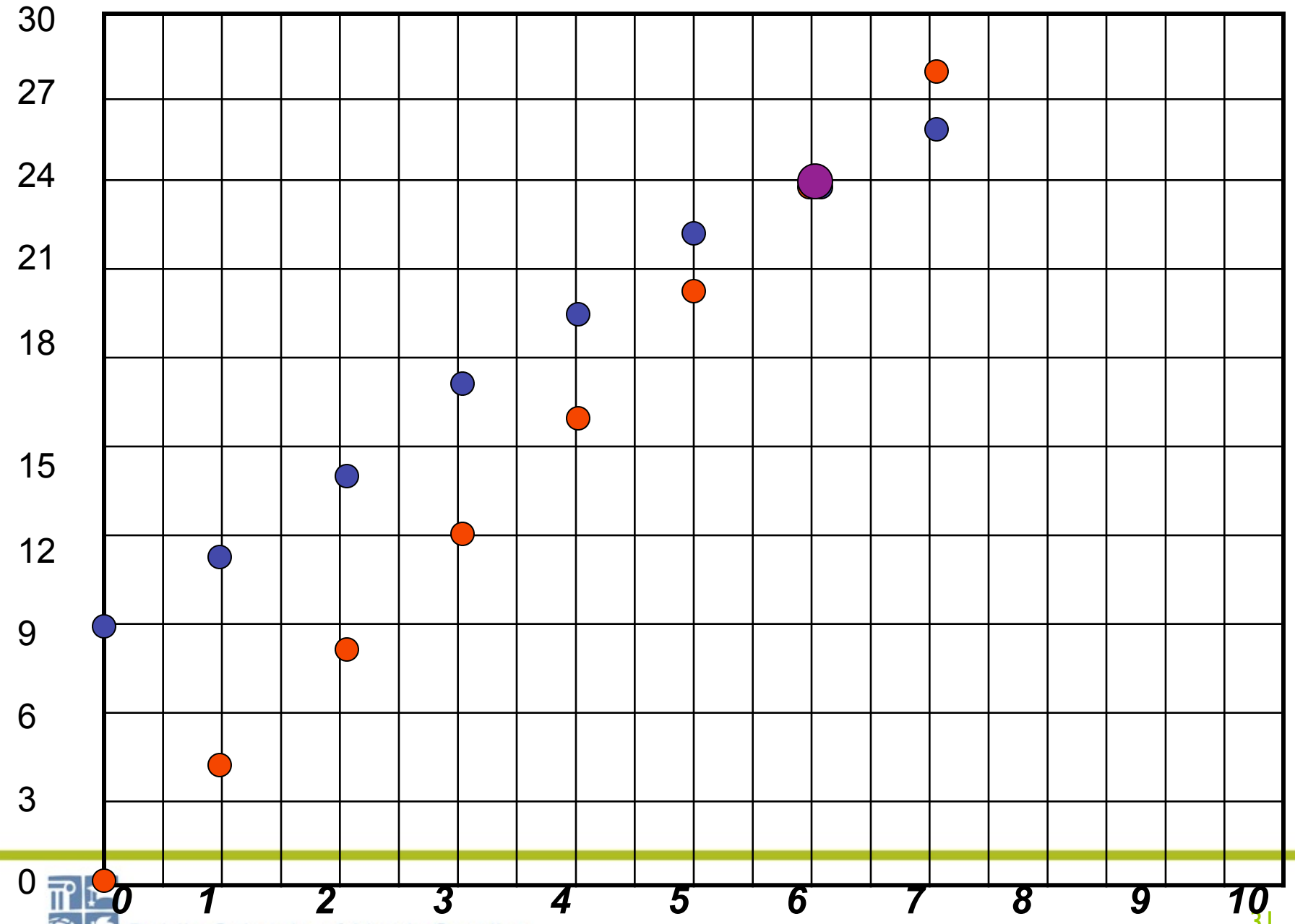
Puggly Wuggly



$2.50 per item

Fancy Foods

You can pay now, or society can pay later…
The best food at the worst prices!
All organic food
$4 per item


30
27
24
21
18
15
12
9
6
3
0
0
1
2
3
4
5
6
7
8
9
10

Slope-Intercept Form

$$y = mx + b$$

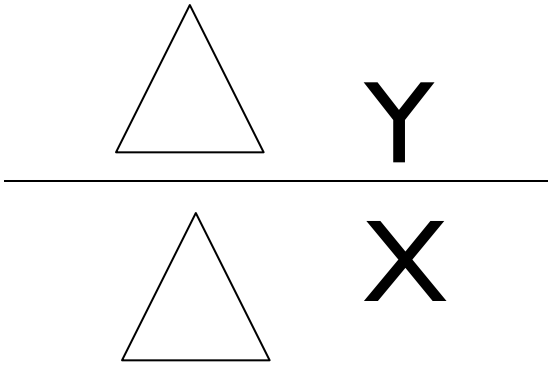
Feeding the Dum Dee (FoA, Intro)

Shopping (FoA, Intro, Alg I)

$$y = \begin{pmatrix} rate\ of \\ change \end{pmatrix} x + \begin{pmatrix} initial \\ amount \end{pmatrix}$$

Given two points, find the equation of the line…

$$\frac{Y_1 - Y_2}{X_1 - X_2}$$

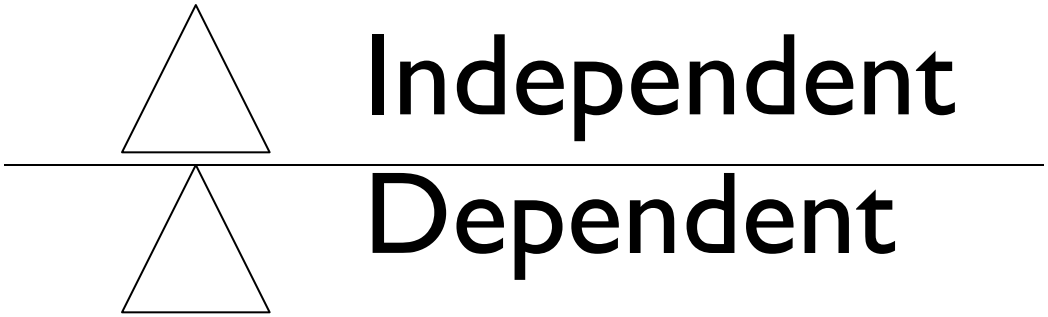


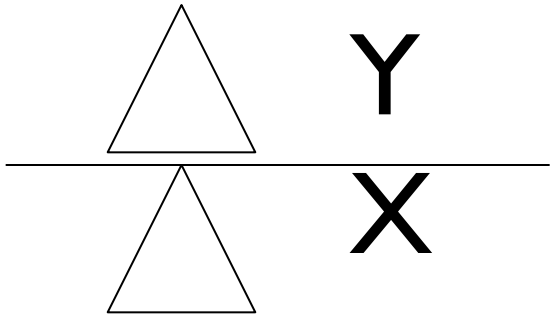
How are your students Processing this?

Why the Y?

$$\frac{\text{Cost}}{\text{Item}}$$

Cost ***per*** Item





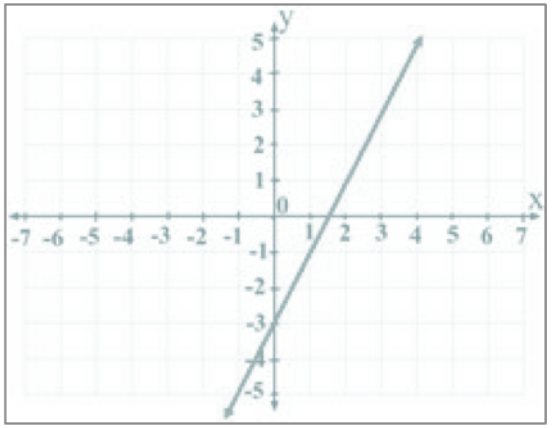
(5,17)
(6, 20)

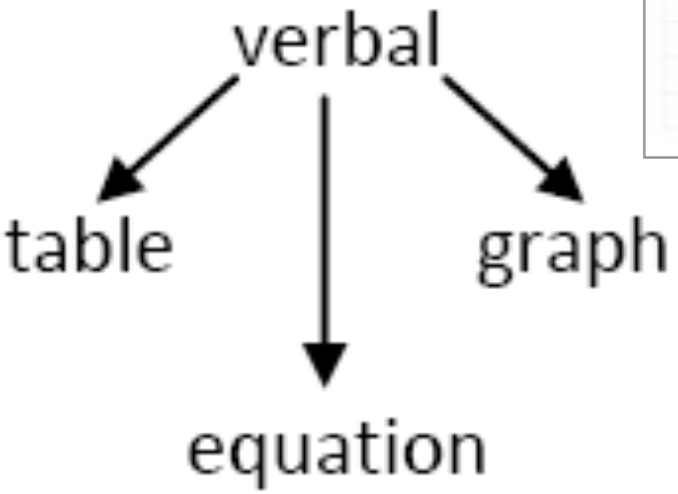
Write an Equation and a STORY to describe the line

Write the Equation Given Two Points

$$y = mx + b$$

x	y
–2	3
–1	1
0	–1
1	–3

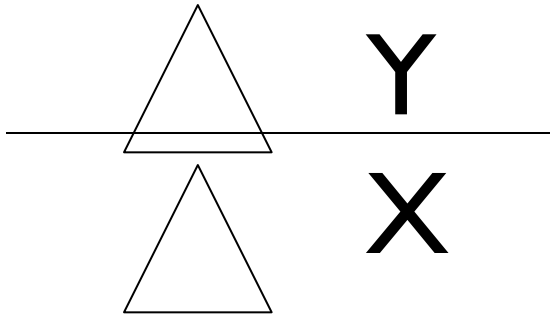




20
18
16
14
12
10
8
6
4
2
0

If students have built understanding through the conceptual/shopping model
how will they see these points now?

(5, 17)
(6, 20)



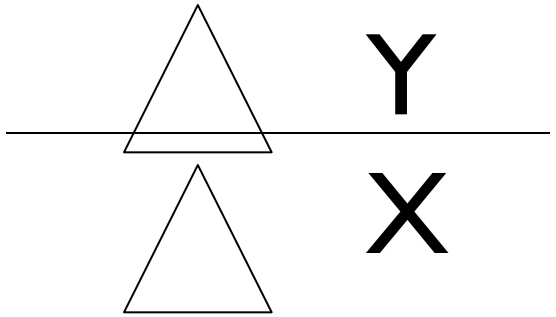
0 1 2 3 4 5 6 7 8 9 10

20
18
16
14
12
10
8
6
4
2
0

If students have built understanding through the conceptual/shopping model
how will they see these points now?

(6 items, $20)

(5 items, $17)



0 1 2 3 4 5 6 7 8 9 10

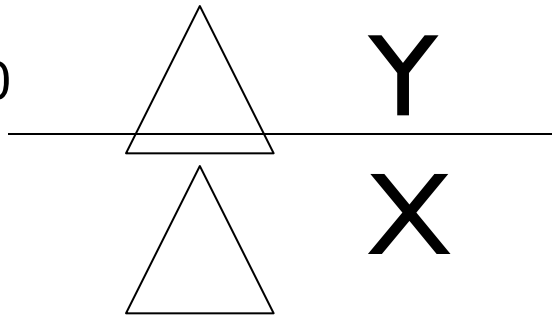
Now I can see it!
That just means:
How much does
the cost change
If I buy one more item?

(6 items, $20)

(5 items, $17)

(5, 17)
(6, 20)

Let's see if I
can figure out
the
Equation
of the
Store/Story
from the
detail of these
two shoppers!



Determine the equation of a line with these two points

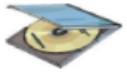
(3, 18)
(5, 28)

Seeing
Structures

Creating
Equations

Reasoning
with
Equations

Verbal:



You and your friends want to record a CD. The company you want to hire charges $300 for making the master and designing the art for the cover. It charges $2.50 for burning each CD. Use the function rule P(c)=300 + 2.5c. Make a table of values and a graph.

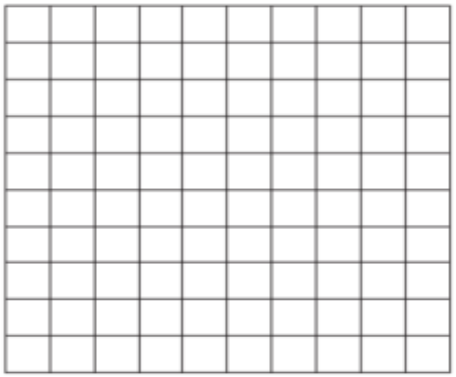
Algebraic

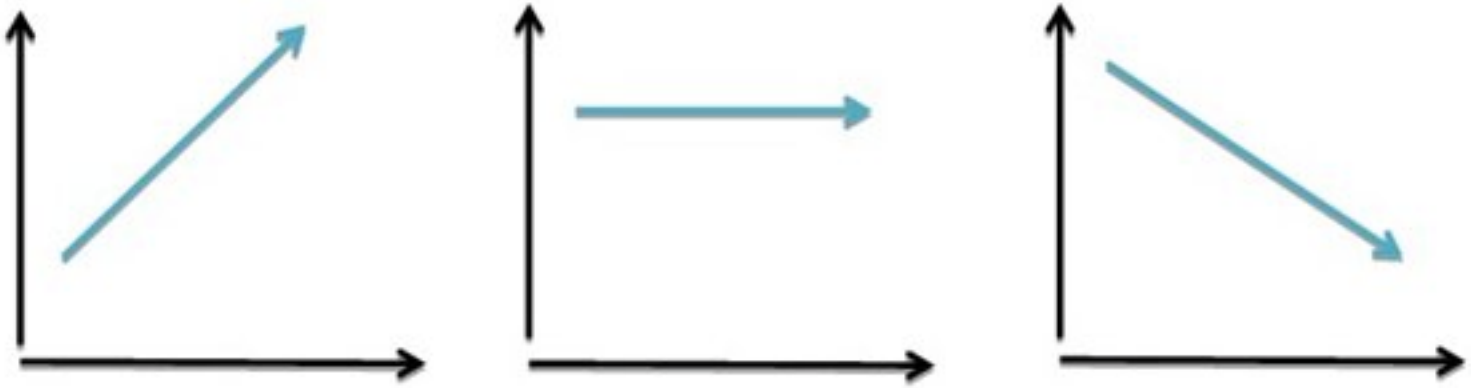
_____

Table

_____(x)	Function: _____	_____ (y)

Graph



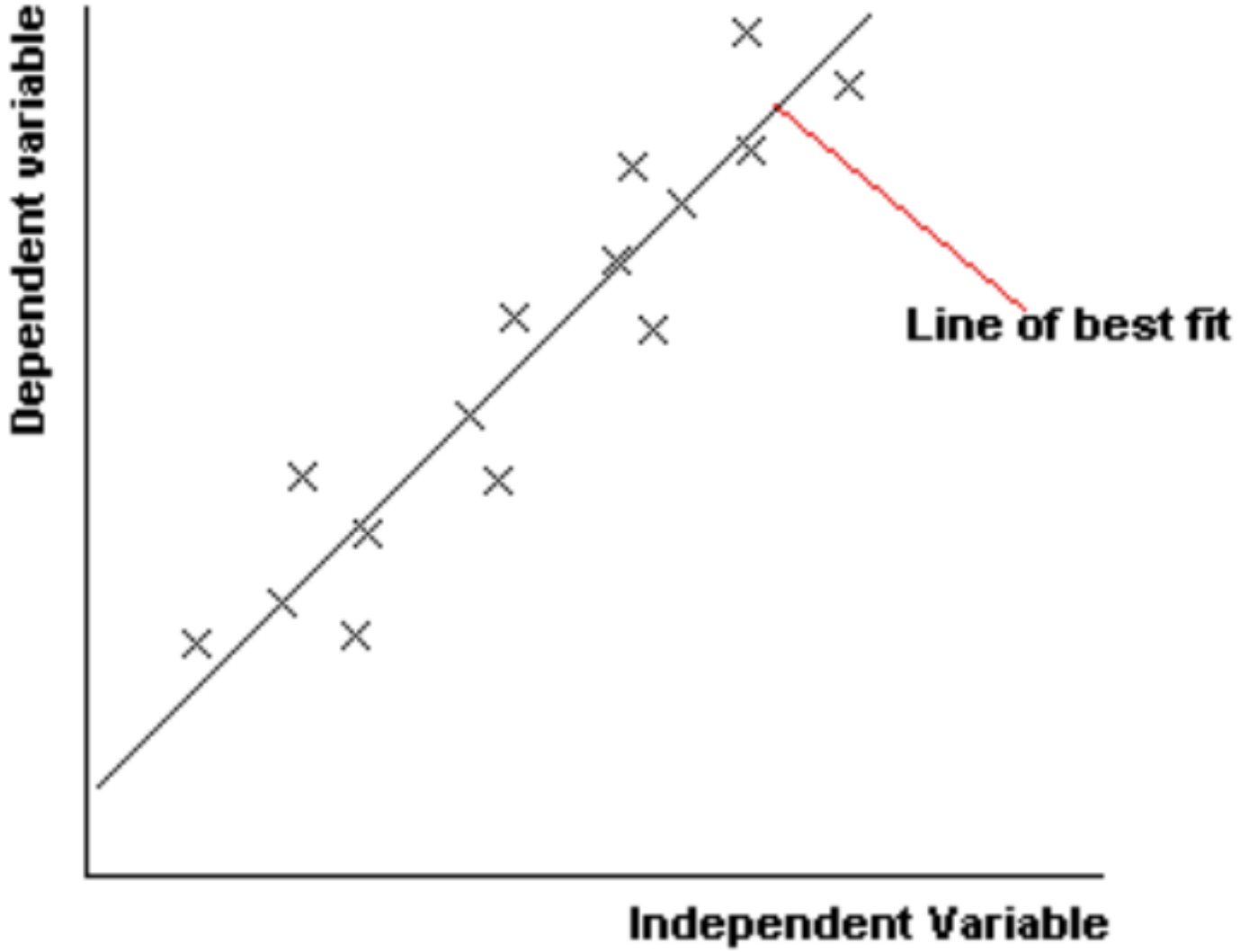


Pearson's correlation coefficient **"r"** is **defined** as the **covariance** (it's a measure of how much two variables change together) of the two variables **divided** by the **product of their standard deviations**.

$$r = \text{Covariance} / (\text{S.D.1} * \text{S.D.2})$$

$$r = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}$$

http://www.mbaskool.com/business-concepts/statistics/8581-pearsons-co



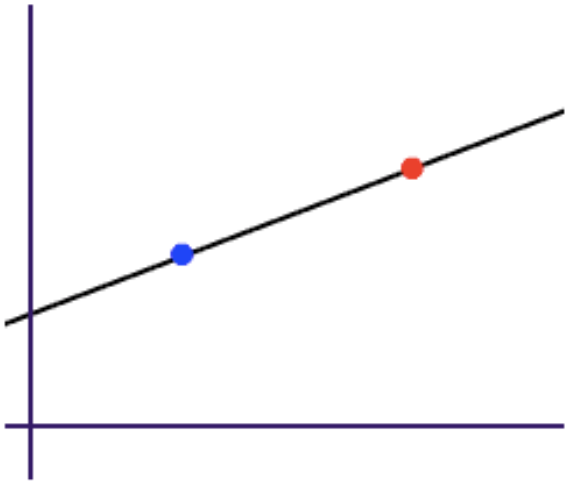
http://www.mathsrevision.net/gcse-maths-revision/statistics-handling-da

http://www.mathscoop.com/calculus/what-is-calculus.php

Differential Calculus

Algebra vs Calculus

In Algebra, we are interested in finding the slope of a line.



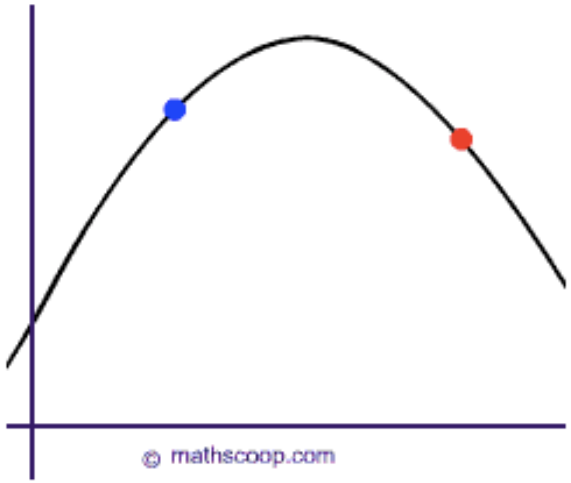
The slope of the line is the same everywhere. The slope is constant and is found using

$$\frac{\Delta y}{\Delta x}$$

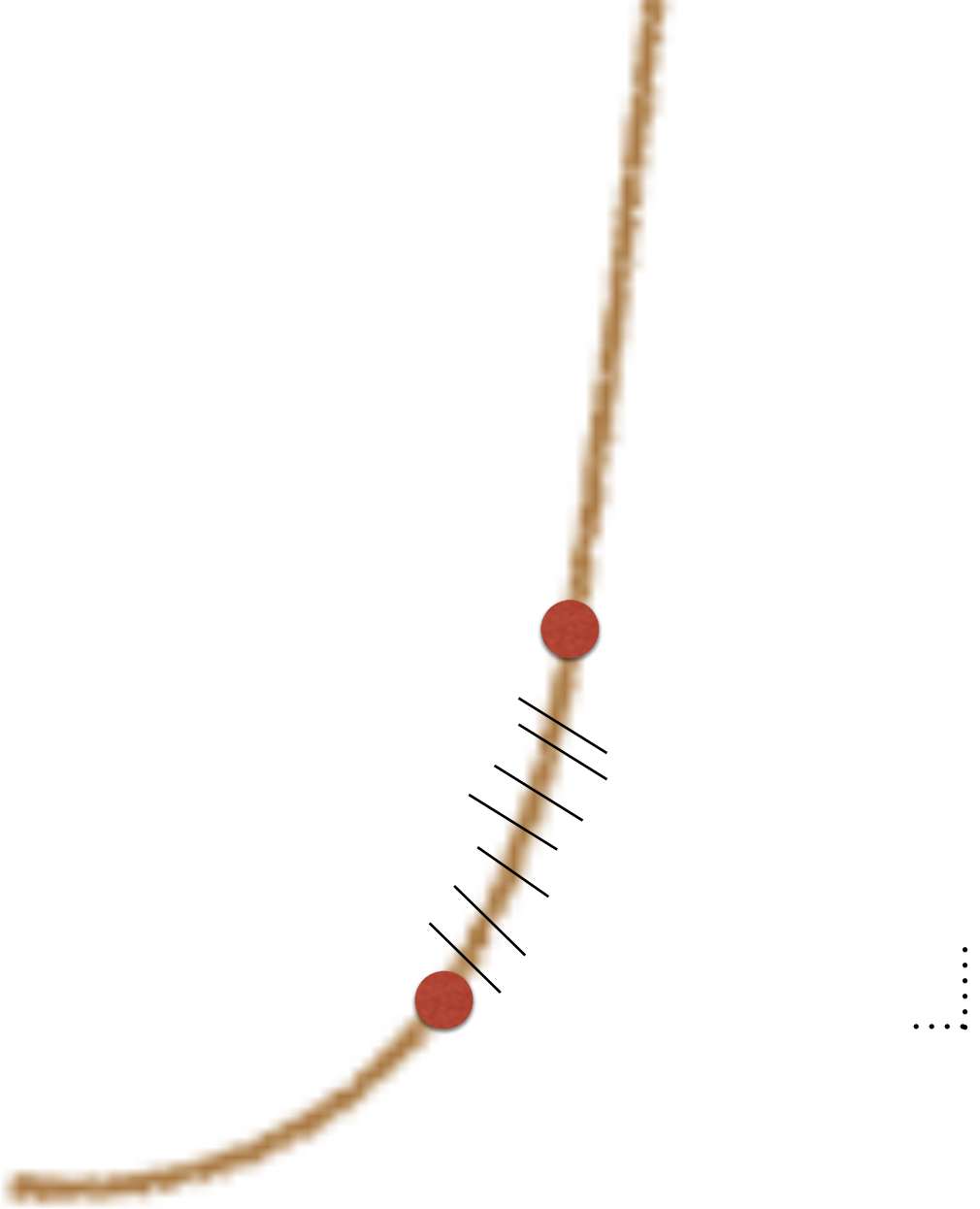
or

$$\frac{rise}{run}$$

In Calculus, we are interested in find the slope of a curve



The slope varies along the curve, so the slope at the red point is different from the slope at the blue point. We need Calculus to find the slope of the curve at these specific points



Make the space between the two points so <u>infinitesimally</u> small…

http://test.metromomsblog.org/?cat=62

We find the <u>Limit</u> as the distance between the two points approaches 0…

Tangent line: The Slope at that point!

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http://www.udlcenter.org/aboutudl/take_a_tour_udl

Reflection on Two Themes

1. Central Conceptual Structure
2. Task Analysis: How does **our understanding** of Algebra affect how we **instruct**, what we **monitor,** what we **assess**?

Questions?

Valerie Faulkner
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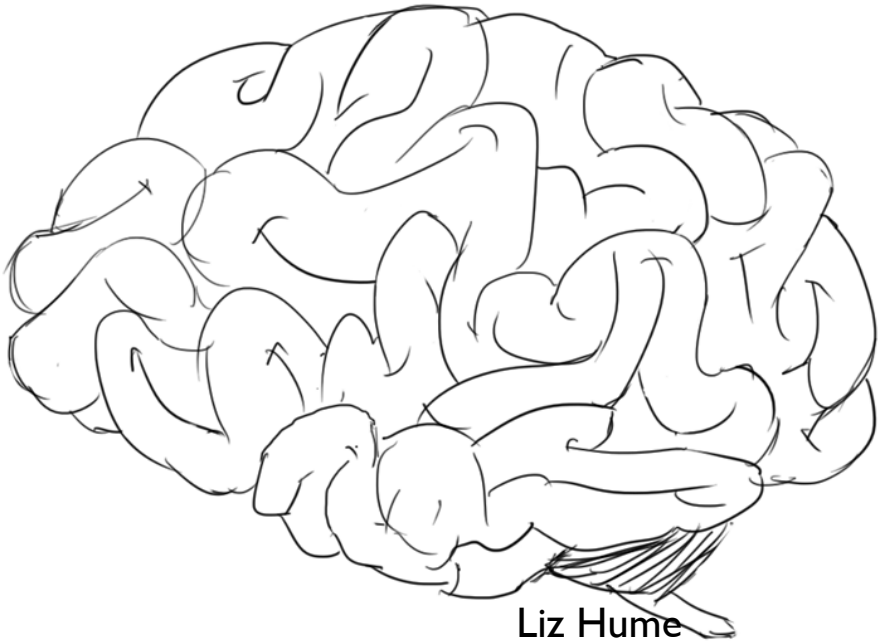
valeriefaulknermathclub.com
valerie_faulkner@ncsu.edu



Liz Hume