Fractions as easy as 1, 2, 3

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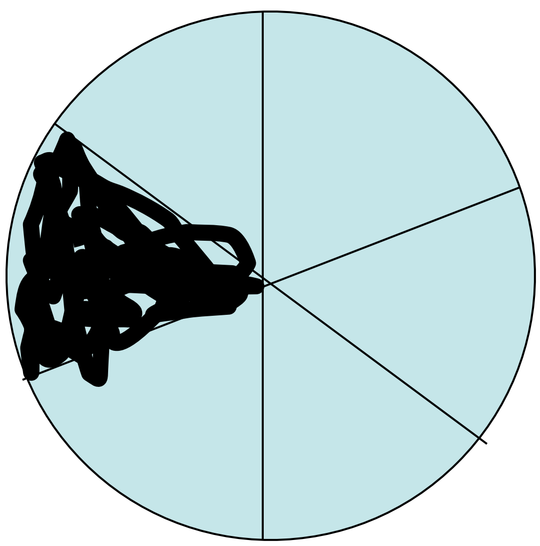
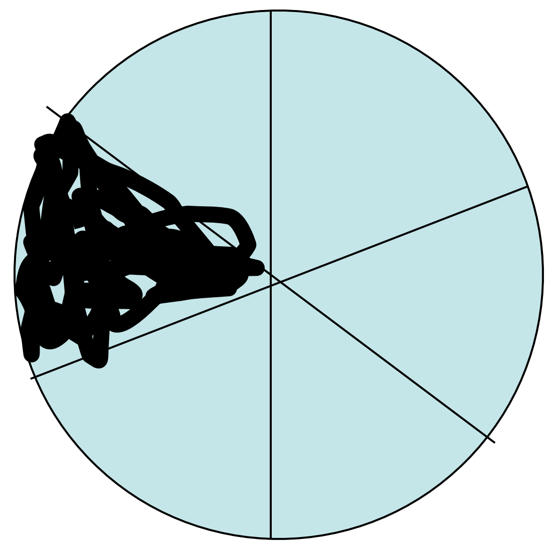
How do you explain this to a student?



1/6 + 1/6 = 2/6



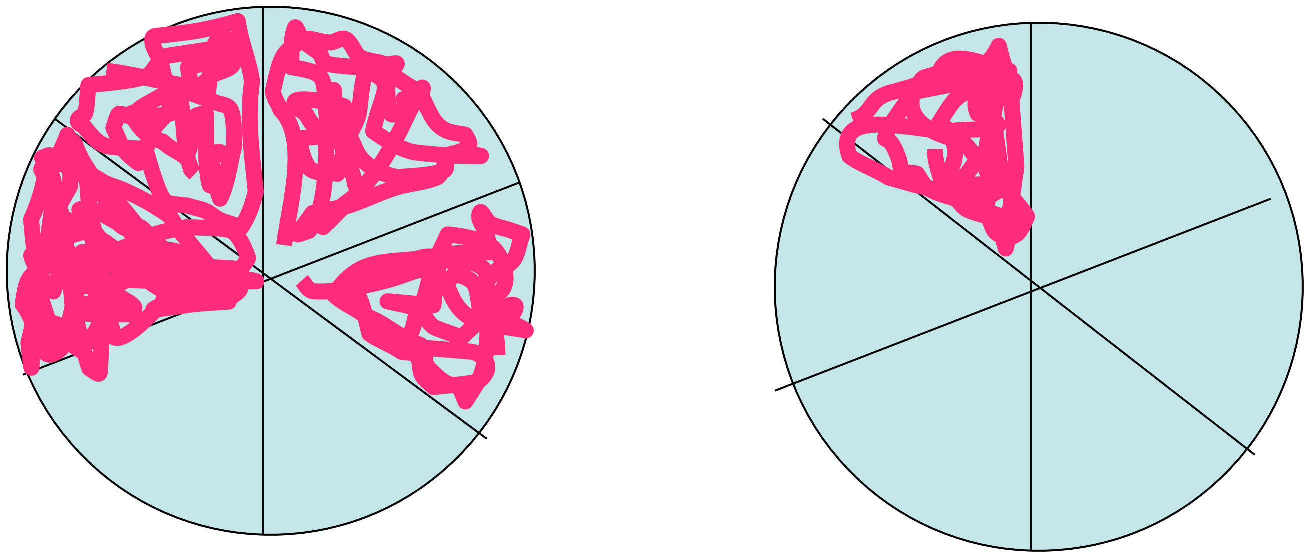
1/6 + 1/6 = 2/12

Game from SRA Real Math

I roll 5/6

What should I do with my 5/6ths?

4/6 + 1/6 = 5/6



What ‘marks’ do you need to hit for fractions?

Guiding your own instruction through considering the key ‘aha’ moments and content connections students need to hit, play with, and own.

Assessment Drives Instruction: Fractions

- Do not ask ‘does the student get the right answer’?
- Ask instead – is the student having the right ‘aha’ moments.

- Don’t look at the kid’s paper…
- Look into the kid’s brain!

How do you introduce **Operations on Fractions?**

- What do you see as the key marks a student needs to hit as you work on virtually any introductory fraction lesson?

- COUNT ⟷ same unit   Wholes   quarters

- ADD + SUB   Just counting   SAME UNIT SIZE

- $1\frac{3}{4}$     $7/4$

MIXED UNIT NUMBER

DECOMPOSE UNITS   Need fraction in form we want it in

[] ⁚   ACCESS to the ones I need

$1\frac{2}{3}$
$-\ 5$

(Next YEAR different unit SIZE)

$1\frac{1}{3} - 2/3$

$\frac{4}{3} - \frac{2}{3}$

$2/3$

Key Aha's for fractions…That will never go away no matter who you are teaching, where you are teaching it, and with what textbook/curriculum.

Intro to Fractions AND

Adding and Subtracting Fractions

- Counting
- Unit Size and "Denominating to Count"
- Decomposing a higher unit value/Mixed numbers
- What do you do if you don't have the same unit size?

Marks 1 & 2 – Counting, unit size and denominating to count

- Do your students really understand counting?
- Do they understand that adding (and its reciprocal subtracting) are just counting?
- Do they understand the connection between Counting and Unit size?

What is 1?

“The idea of number is based on a division of the world into
two levels:

the same and different”

- Denis Guedj

from Guedj, 1999, Numbers: the Universal Language
and from
Andy Warhol, 1963, Liz Taylor 10 times

In other words…

- 1 what?

One is one, or is it?

naming units

de- "completely" +
nominare "to name”

de-NOM-inations…

de-NOM-inators…

and naming units (de-NOM-inations)...

3 bears and 2 bears equals 5 bears

3 ducks and 2 ducks equals 5 ducks

3 bears and 2 ducks equals...

5 berucks?

and naming units (de-NOM-inations)...

3 animals (bears) and 2 animals (ducks) equals...

5 ANIMALS

Connections Across Curriulum
Unit Size

3 ones and 2 ones

3 tens and 2 tens

3 tens and 2 ones

3/6   and 2/6

3/6 and 2/5

3X and 2X

3Y and 2Y

3X and 2Y

Extension Question or Prompt

In algebra you don't always know everything about what you are adding. Sometimes you will have unknowns. For instance, you might have 3x's and 5y's. This would be written 3x and 5y. I don't know what the x is, but I know I have three of them. I don't know what the y is exactly, but I know I have 5 of them.

Use the logic we just used with fractions to explain to someone how you would handle the following expressions:

$3x + 2x$

$5x - 2x$

$3y + 2y$

$3x + 2y$

- 4th grade

Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

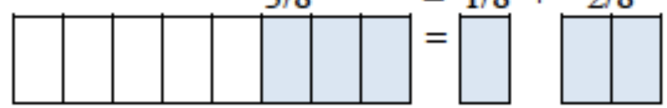
Example:

1 ¼ - ¾ = □ 4/4 + ¼ = 5/4 5/4 – ¾ = 2/4 or ½

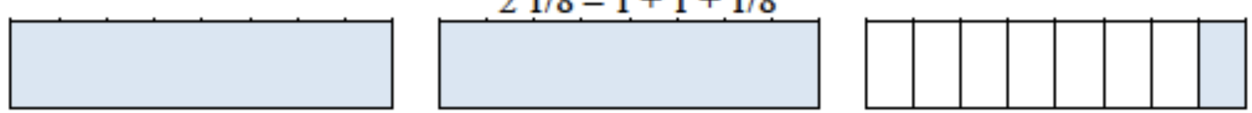
Example of word problem:
Mary and Lacey decide to share a pizza. Mary ate 3/6 and Lacey ate 2/6 of the pizza. How much of the pizza did the girls eat together?
Possible solution: The amount of pizza Mary ate can be thought of a 3/6 or 1/6 and 1/6 and 1/6. The amount of pizza Lacey ate can be thought of a 1/6 and 1/6. The total amount of pizza they ate is 1/6 + 1/6 + 1/6 + 1/6 + 1/6 or 5/6 of the whole pizza.
Students should justify their breaking apart (decomposing) of fractions using visual fraction models. The concept of turning mixed numbers into improper fractions needs to be emphasized using visual fraction models.
Example:

3/8 = 1/8 + 1/8 + 1/8

3/8 = 1/8 + 2/8

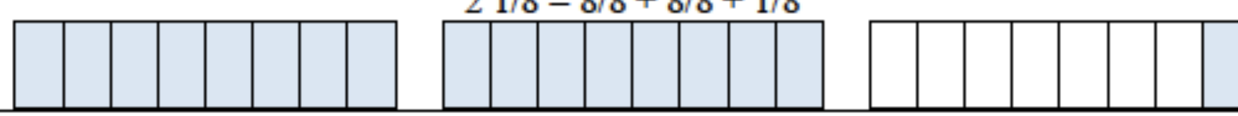


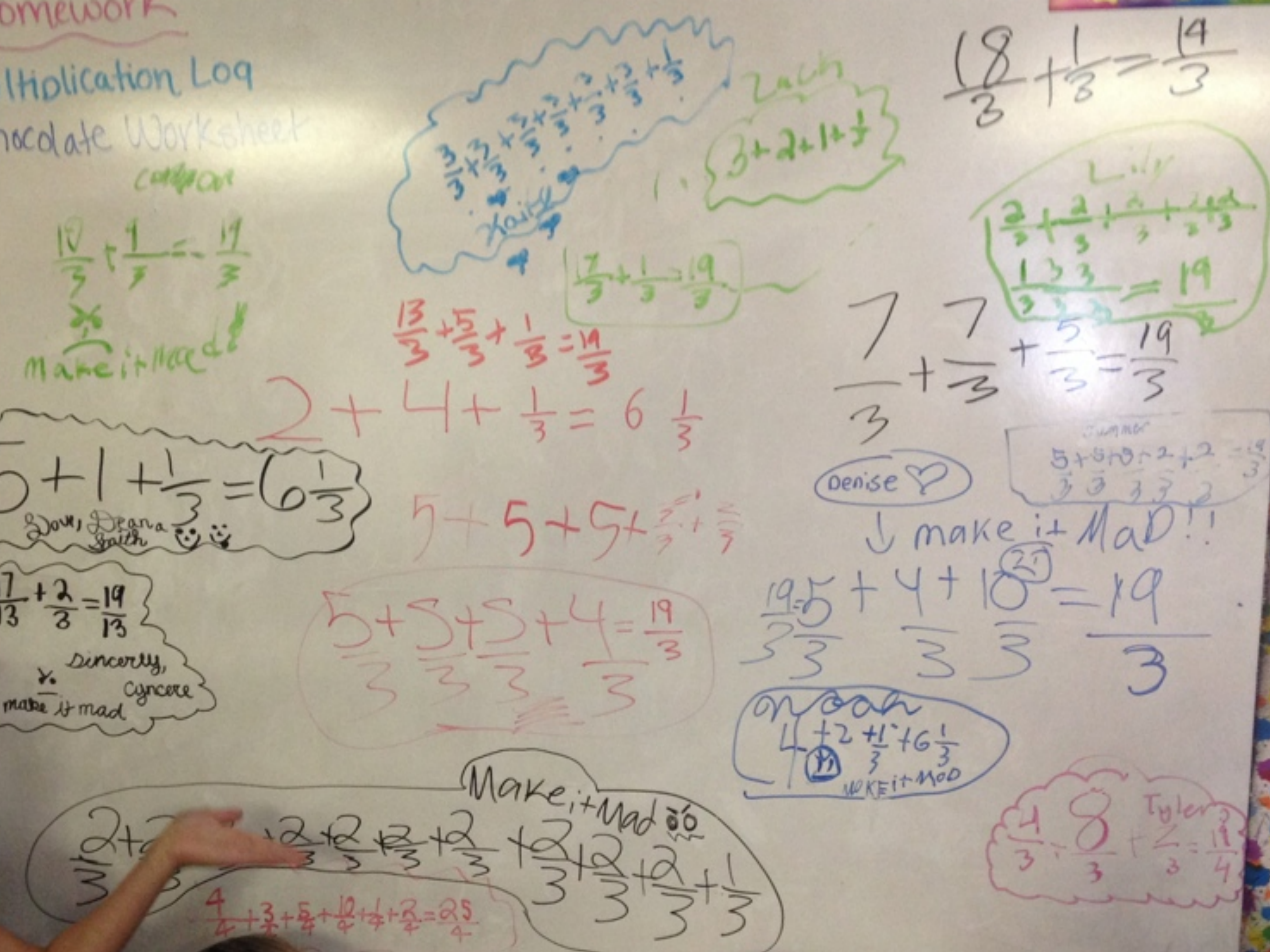
2 1/8 = 1 + 1 + 1/8

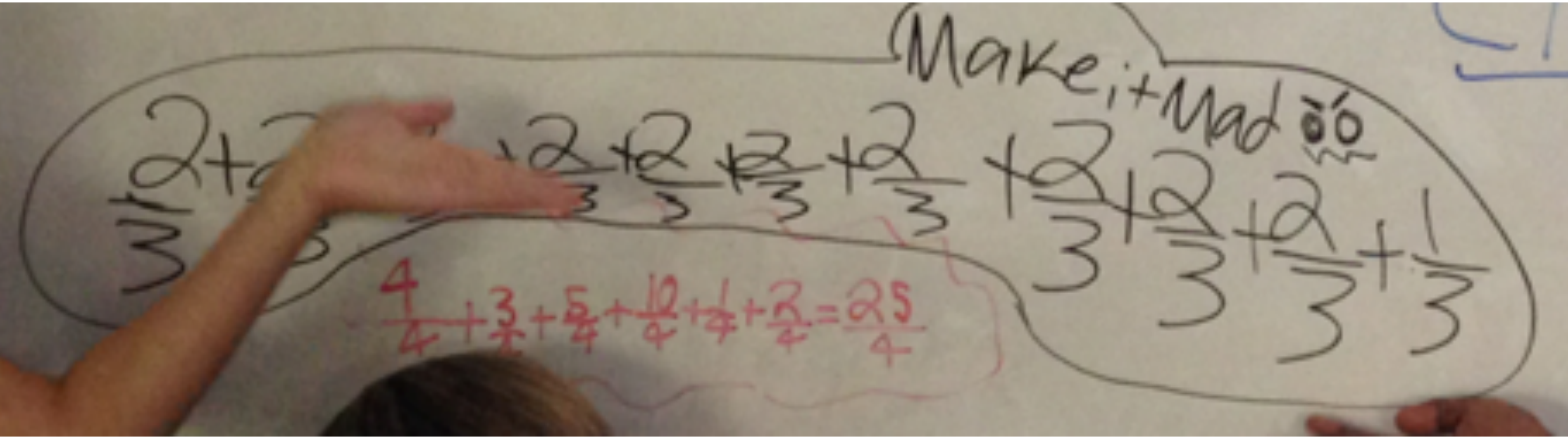


or

2 1/8 = 8/8 + 8/8 + 1/8




omework
ltiplication Log
ocolate Worksheet
Zach
3+2+1+1/3
Lily
Denise
make it MaD!!
Make it Mad
sincerely,
Cyncere
make it mad
Noah
MAKE it MAD
Tyler
18/3 + 1/3 = 19/3
7/3 + 7/3 + 5/3 = 19/3
2 + 4 + 1/3 = 6 1/3
5 + 1 + 1/3 = 6 1/3
5/3 + 5/3 + 5/3 + 4/3 = 19/3
13/3 + 5/3 + 1/3 = 19/3


Make it Mad
4/4 + 3/4 + 5/4 + 10/4 + 1/4 + 2/4 = 25/4

Mark 3 —
Mixed Numbers/Decomposing a higher unit value

K-3

15 - 8

How do you teach facts within 20?

4-12

$1\ 3/8 - 5/8$

How do you teach problems such as the above?

Is it possible that these two situations are, essentially, the exact same problem?

$15 - 8$

$1\ 3/8 - 5/8$

Build on understanding of different forms of value
Decompose a higher unit value at the symbolic level…

$$\begin{array}{r} 0\ \ 15 \\ \not{1}5 \\ -\ 8 \\ \hline 7 \end{array}$$

0 8/8 and 3/8

~~1~~ 3/8

\- 5/8

Decomposing a higher unit value into a lower unit…

using unit size to change the form of the number

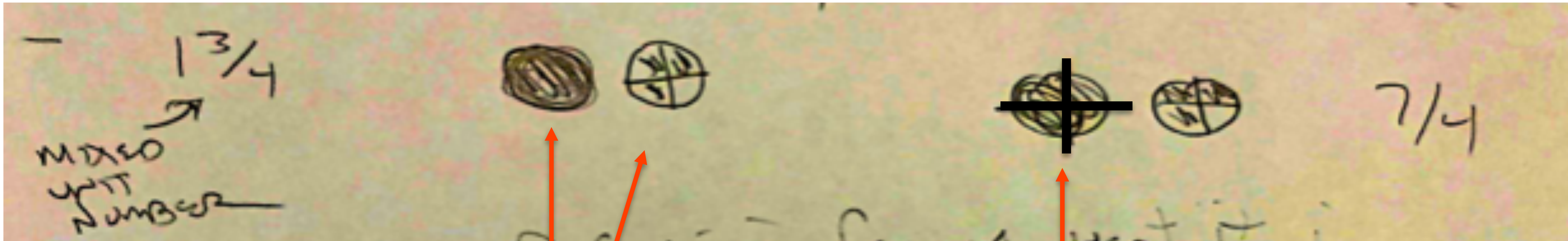
How do college students convert mixed numbers?

- They multiply the denominator by the whole number and then they add the numerator.
- Less than half of my class last year knew *why* they ‘did’ that.
- Their brain was empty…

$1\ 3/4$

$1 \times 4 + 3$

$7/4$



One whole & three quarters

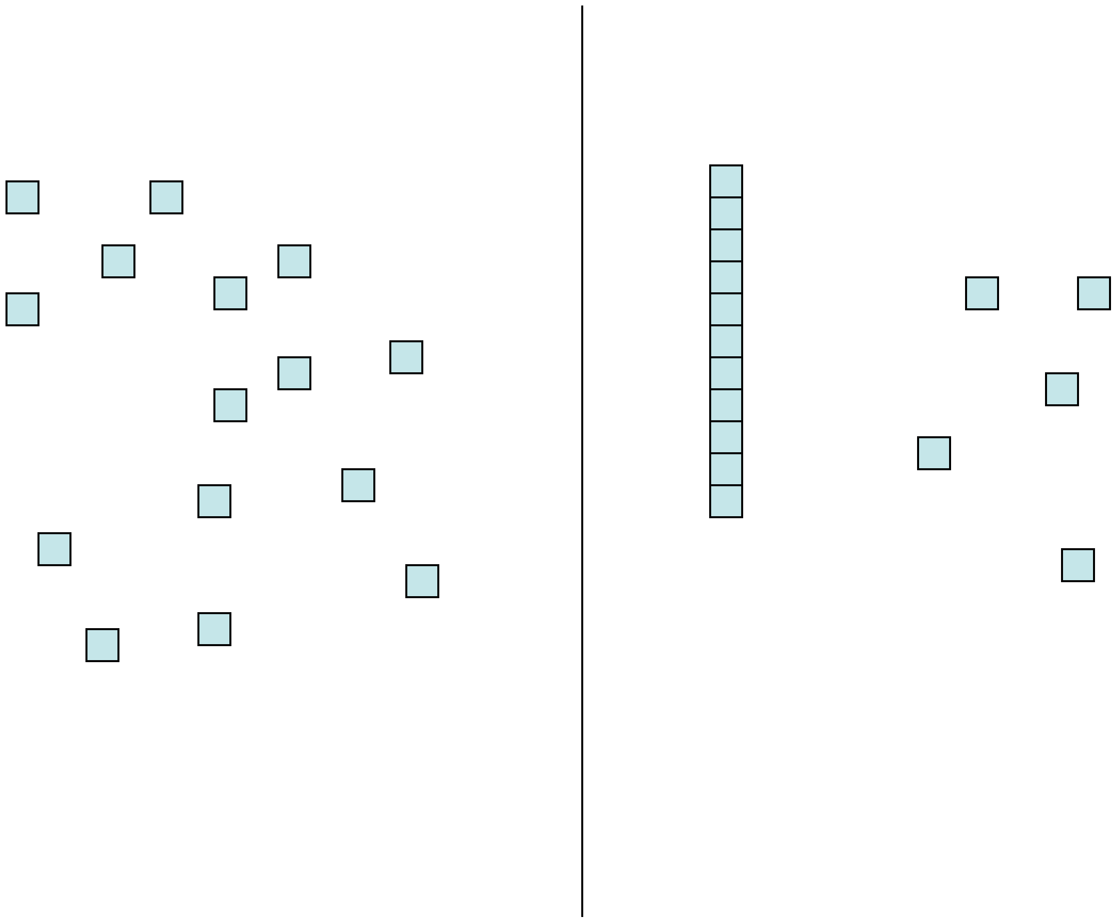
1 x 4 is the *ACT* of chopping my
One whole into fourths

Decomposing the higher unit value

How about the numeral 11

- How is 11 ‘mixed’?
- It is 11 ones, but we name it in our numeration system as 1 ten and 1 one. We clump into tens.
- For any number in standard form I can think of it clumped in powers of ten or decomposed into smaller units

Different forms can look different yet we Maintain the value of the set What does the brain see?



Mark 4 – What if I don’t have same unit size? Am I stuck?

Number and Operation – Fractions[1] 4.NF

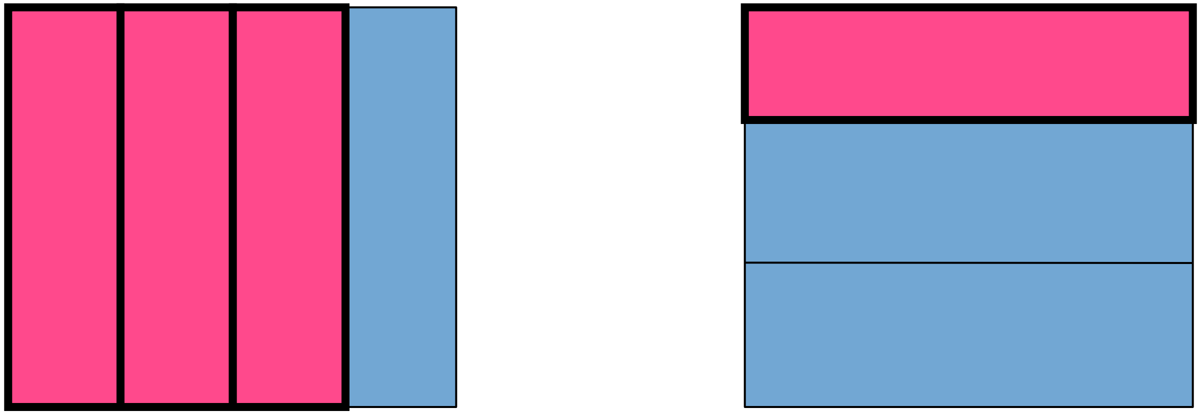
Common Core Cluster

Extend understanding of fraction equivalence and ordering.
Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **partition(ed), fraction, unit fraction, equivalent, multiple, reason, denominator, numerator, comparison/compare, ‹, ›, =, benchmark fraction**

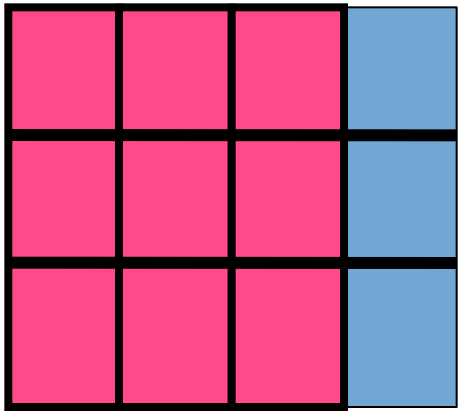
Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
4.NF.1 Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	This standard refers to visual fraction models. This includes area models, number lines or it could be a collection/set model. This standard extends the work in third grade by using additional denominators (5, 10, 12, and 100) This standard addresses equivalent fractions by examining the idea that equivalent fractions can be created by multiplying both the numerator and denominator by the same number or by dividing a shaded region into various parts. Example:  **1/2 = 2/4 = 6/12** Technology Connection: http://illuminations.nctm.org/activitydetail.aspx?id=80
4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are	This standard calls students to compare fractions by creating visual fraction models or finding common denominators or numerators. Students' experiences should focus on visual fraction models rather than algorithms. When tested, models may or may not be included. Students should learn to draw fraction models to help them compare. Students must also recognize that they must consider the size of the whole when comparing fractions (ie, ½ and 1/8 of two medium pizzas is very different from ½ of one medium and 1/8 of one large).

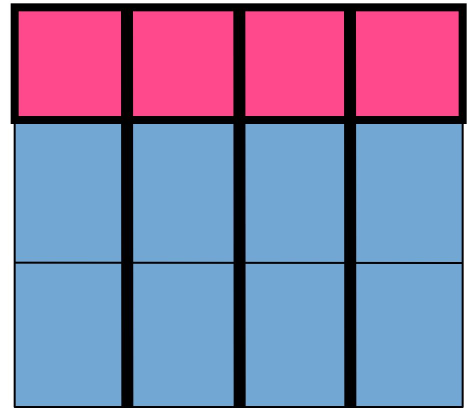
Lee Stiff's Unit Squares for Adding Fractions



¾ + 1/3

We've got a problem in this form—
need the same size pieces to add things…





$\frac{3}{4}$ + 1/3

9/12 + 4/12

Chop up the vertical by the horizontal
and the horizontal by the vertical:
Don’t change the value, just the piece size

Key Aha's for fractions…That will never go away not matter who you are teaching, where you are teaching it, and with what textbook/curriculum.

Multiplying and Dividing Fractions

- The power of the operations: Multiplying is GROUP counting
- How the step up in operation power affects what is happening to the unit.
- What does it mean to 'cancel' out?
- How do you get your like denominator when you multiply?

7 dollars + 5 dollars = 12 dollars

7 dollars * 5 dollars = 35 dollars

$$\frac{35 \text{ dollars}}{7 \text{ people}} = 5 \frac{\text{dollars}}{\text{person}}$$

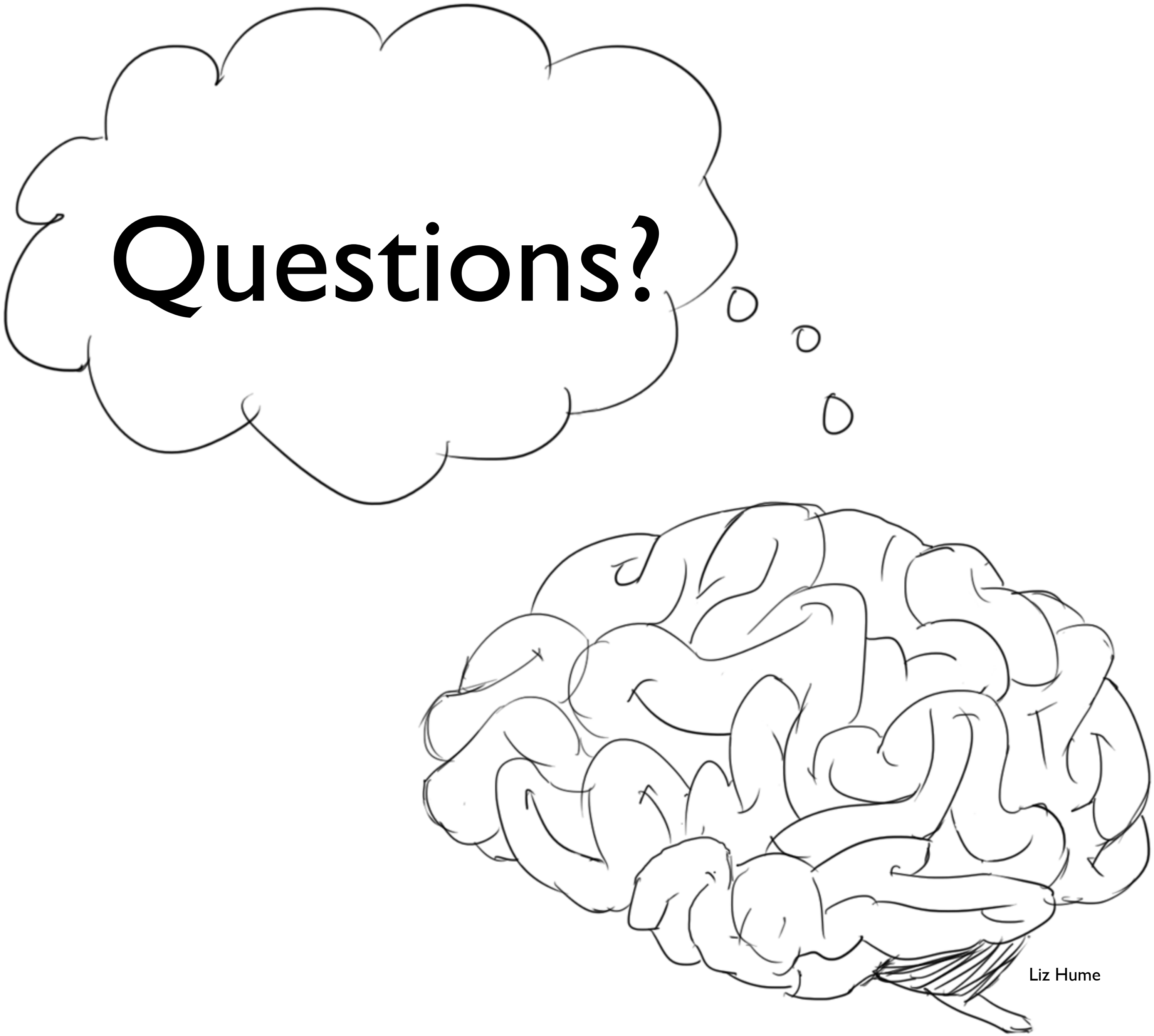
$$\frac{35 \text{ dollars}}{7 \text{ dollars}} = 5^{\text{people}}$$

$$\frac{35\text{ dollars}}{7\text{ dollars}} = 5^{\text{groups}}$$

Why don't I need a common denominator when I multiply?

Because the operation does it for me…

$$\frac{2}{3} \times \frac{3}{5}$$



Liz Hume