Conceptual Algebra Part 1 - Overview and Intro

The Research and the Structure

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Let's try yet another
new thing this year!
Yay! Yay! Yay!

Why?

Seeing **Structures** in Expressions

Creating Equations

CCS Algebra

Reasoning with Equations

NAEP Task

Give the value of y when x = 3.

x	1	3	4	7	n
y	8		11	14	

Source: National Council of Teachers of Mathematics

High Achieving Countries
MAKE CONNECTIONS

U.S.
TEACHES PROCEDURES

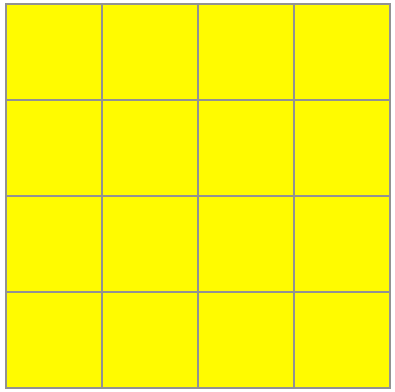
Structures and Connections

What is 4^2 ?

Procedure versus Structure/Connections

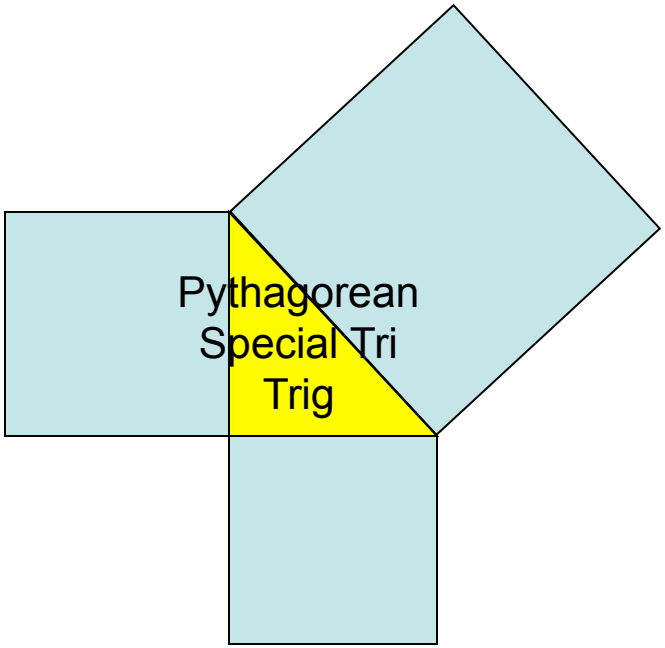
Make a square out of your 4 unit linear side

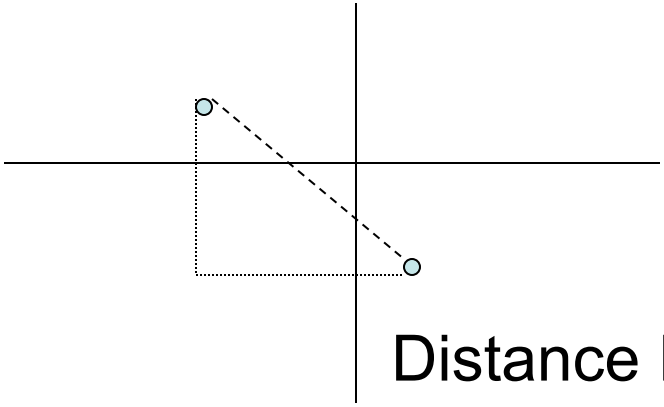
Exponents and CONNECTIONS



Square Roots!
$\sqrt{16} = 4$
The length of one side!

Geometry





Distance Formula

Seeing Structures?
Creating Equations?
Reasoning with Equations?

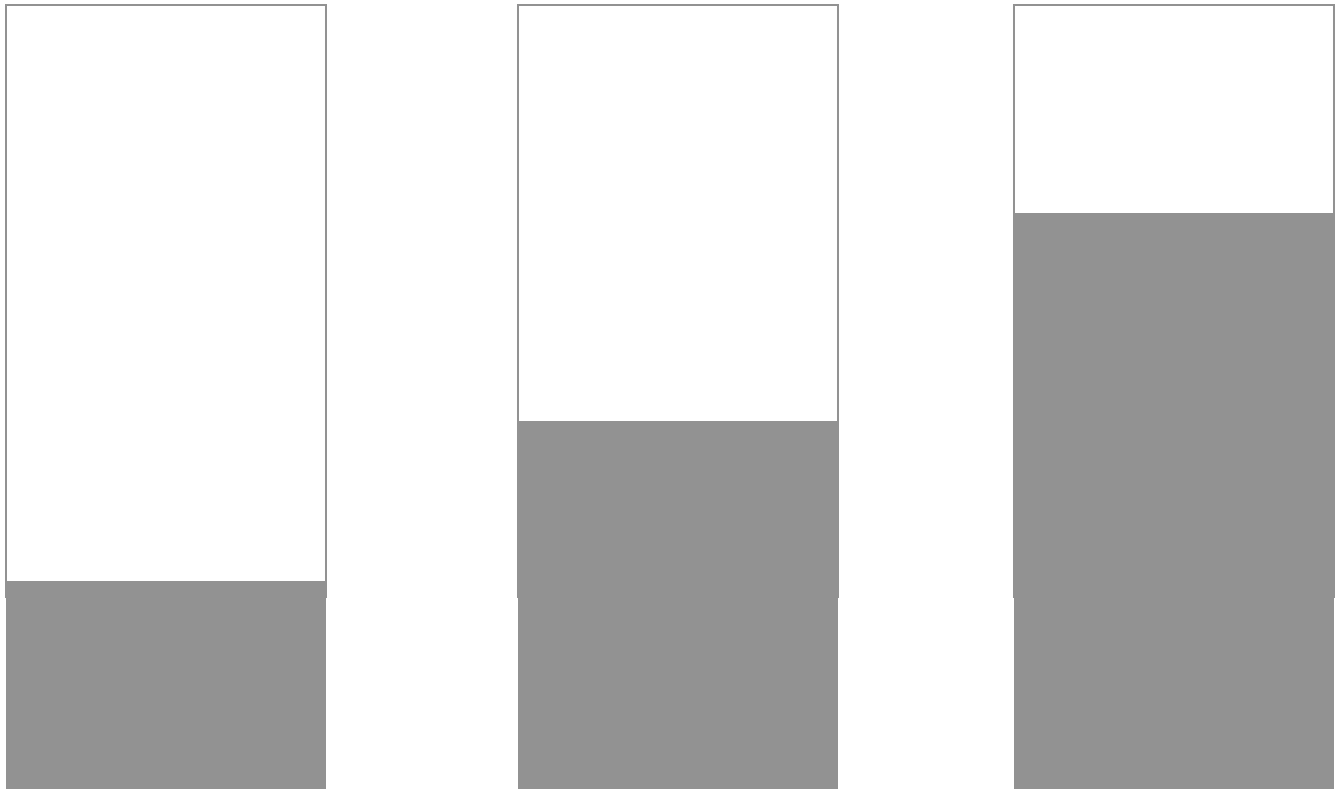
$$3X + 7 = Y$$

Write a story problem for this equation…

Why does this happen?

We teach *digitally* but we ALL have **analog** brains!

The Accumulator Model: Our Analog Brain



See Stanislas Dehaene 1999. The Number Sense

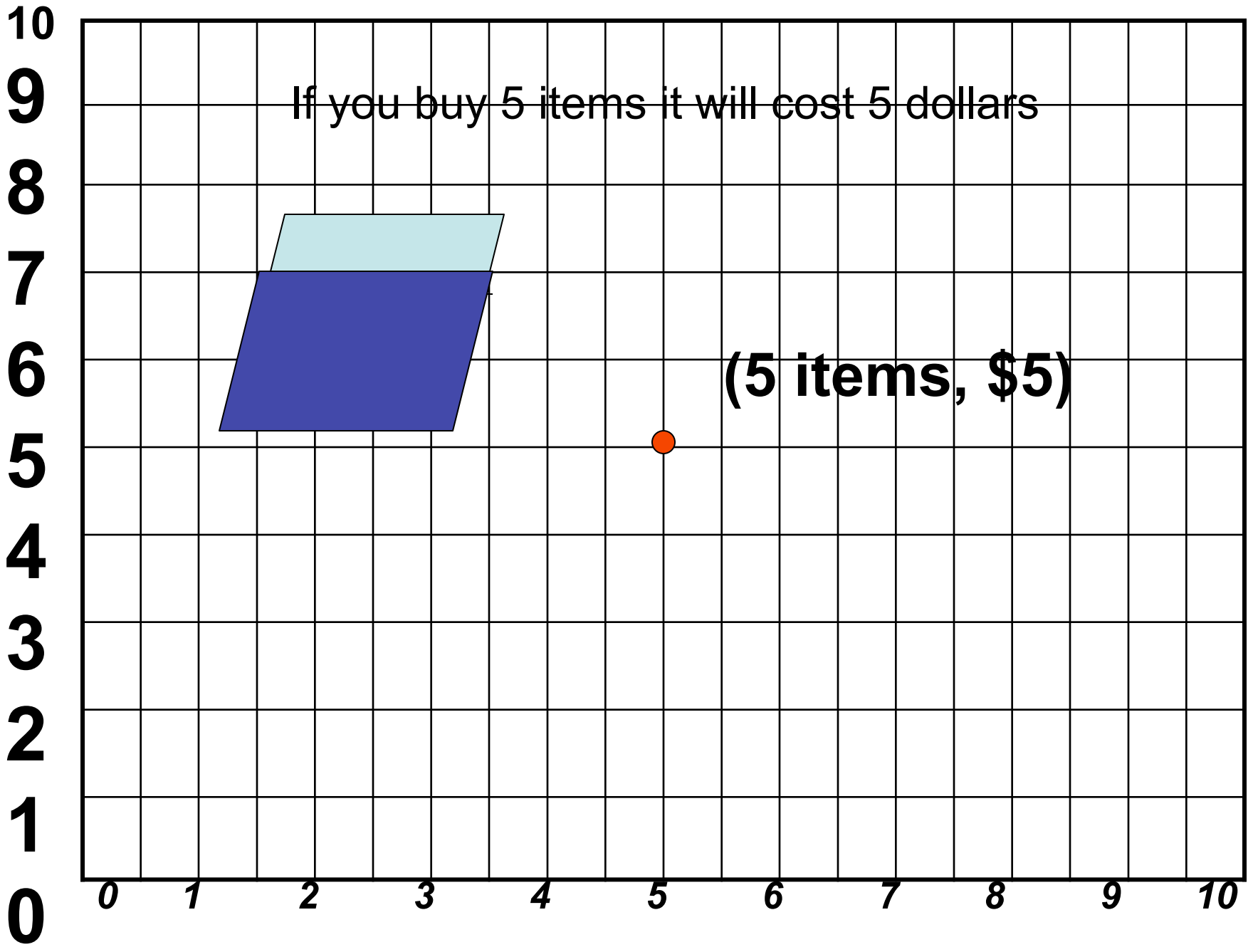
Dollar Deals

No Tax,

No Tricks:

One Dollar per One Item.

Where EVERYTHING is just one dollar per item!


If you buy 5 items it will cost 5 dollars
(5 items, $5)
10
9
8
7
6
5
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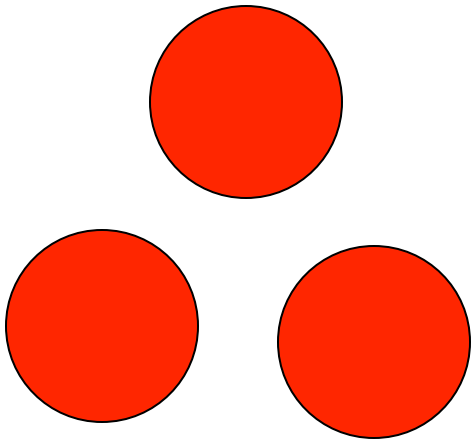
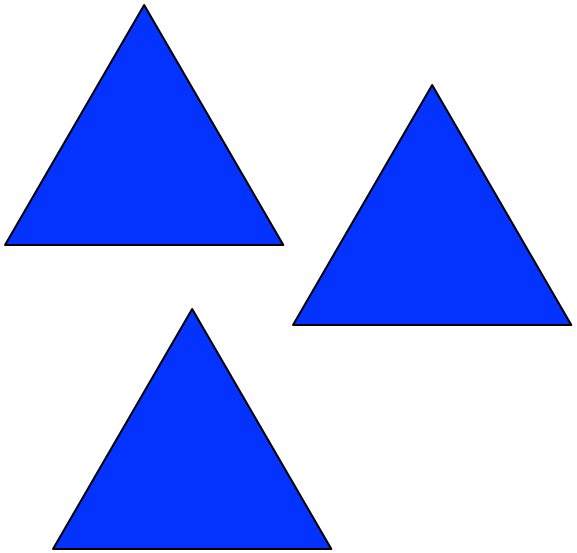
9
10

$$X = Y$$

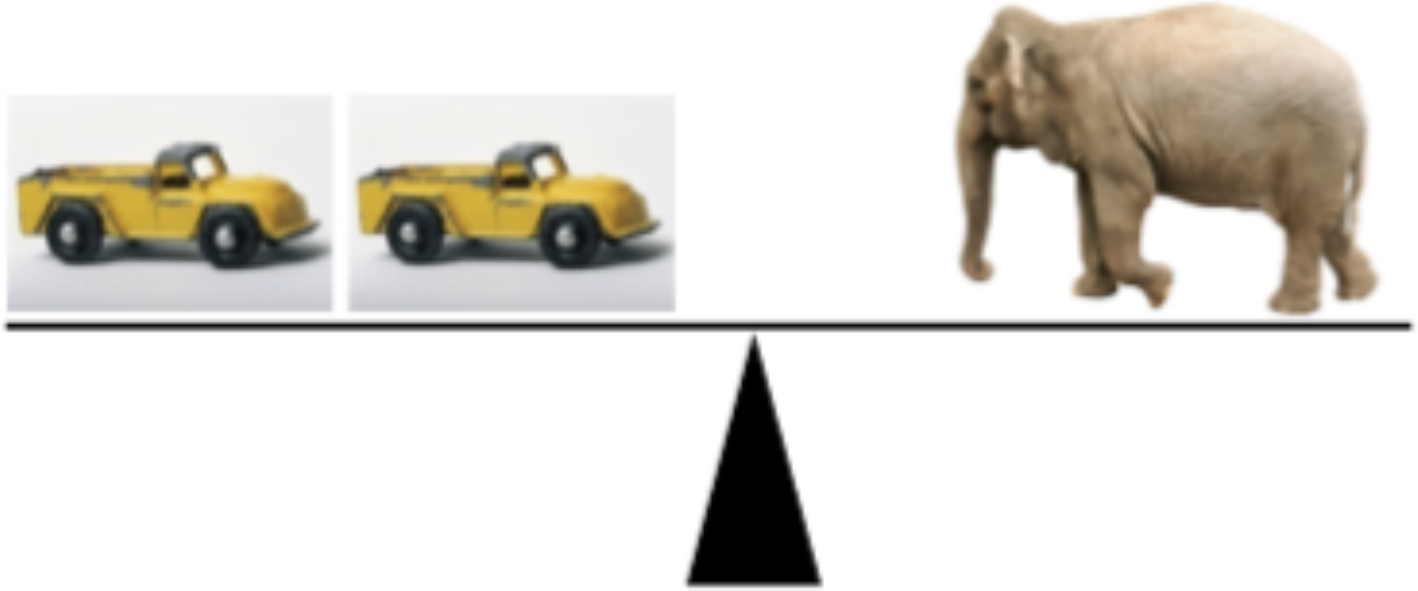
What's the 'bug' in our kids' systems regarding Equality?

"Same As"

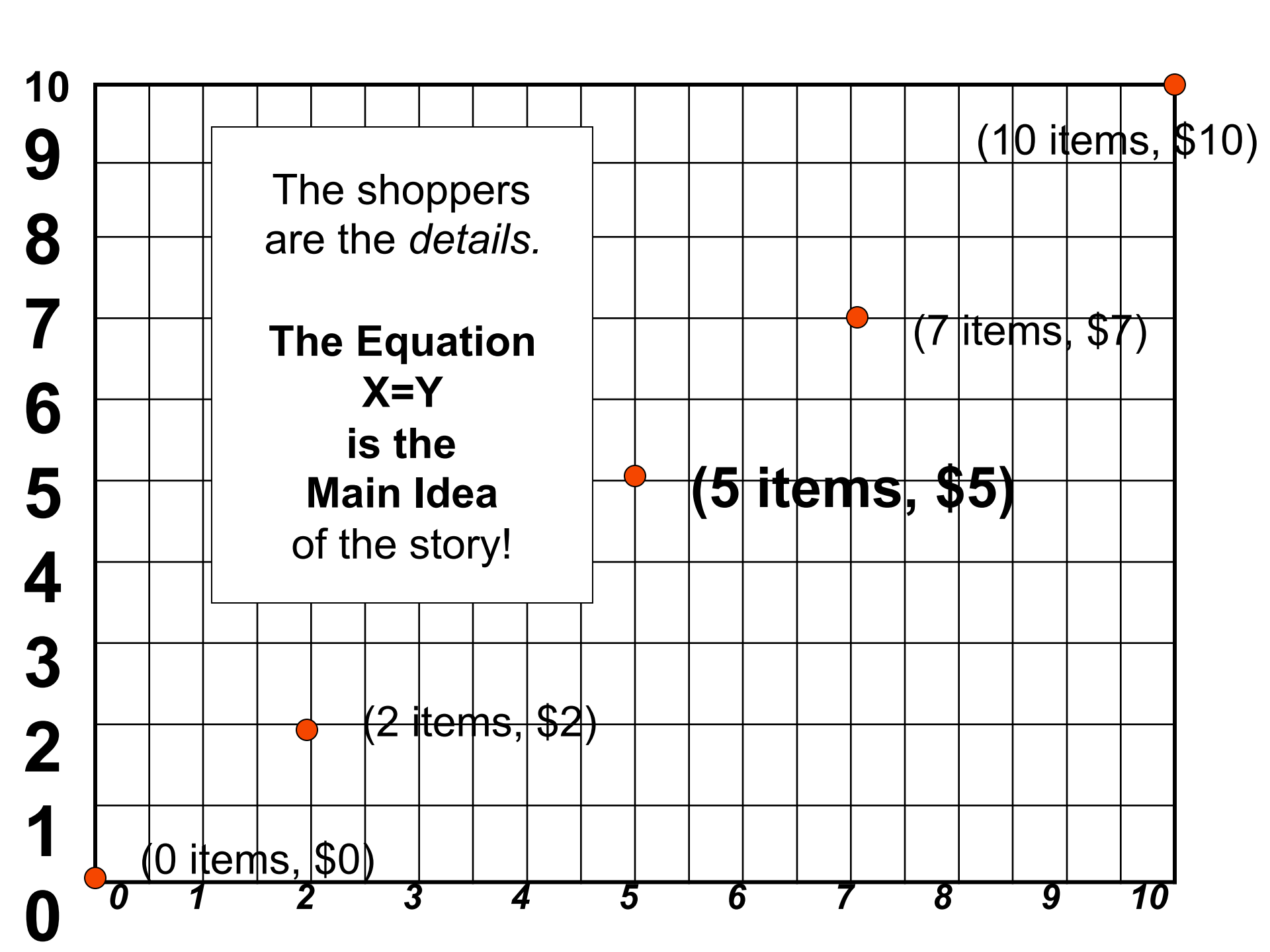




‘Same as’



$$X = Y$$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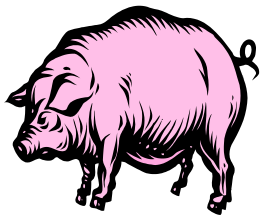
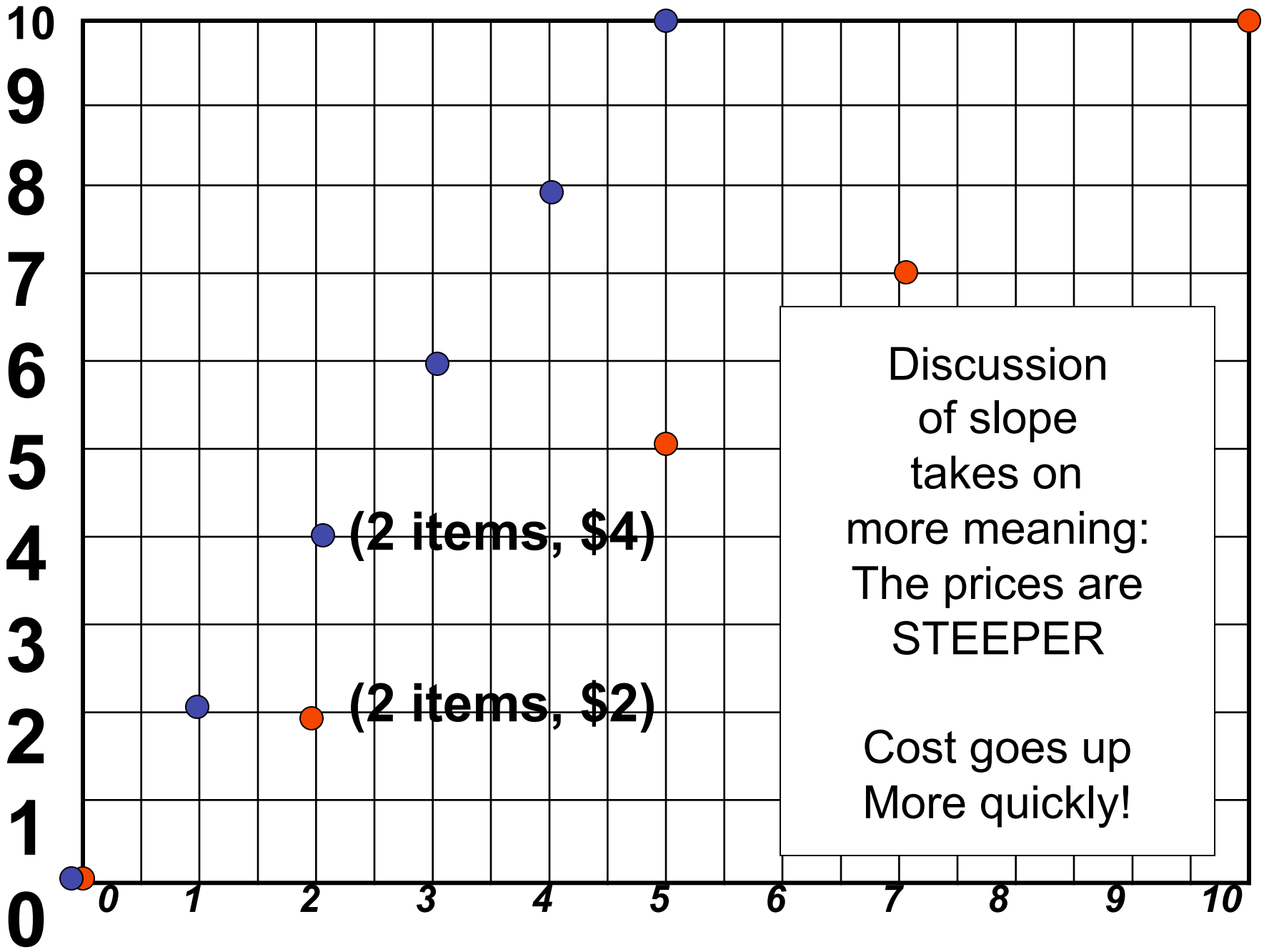
10
The shoppers are the *details.*
The Equation X=Y is the Main Idea of the story!
(10 items, $10)
(7 items, $7)
(5 items, $5)
(2 items, $2)
(0 items, $0)

Puggly Wuggly Super Sale!

All items you can fit in 1 bag:

$2 for each item in bag!

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10
(2 items, $4)
(2 items, $2)
Discussion of slope takes on more meaning: The prices are STEEPER
Cost goes up More quickly!

Verbal:

You and your friends want to record a CD. The company you want to hire charges $300 for making the master and designing the art for the cover. It charges $2.50 for burning each CD. Use the function rule P(c)=300 + 2.5c. Make a table of values and a graph.

Algebraic

Table

_____(x)	Function: _____	_____ (y)

Graph

Shopping Construct for linear algebra…

Basic Structure

Vocabulary with Meaning

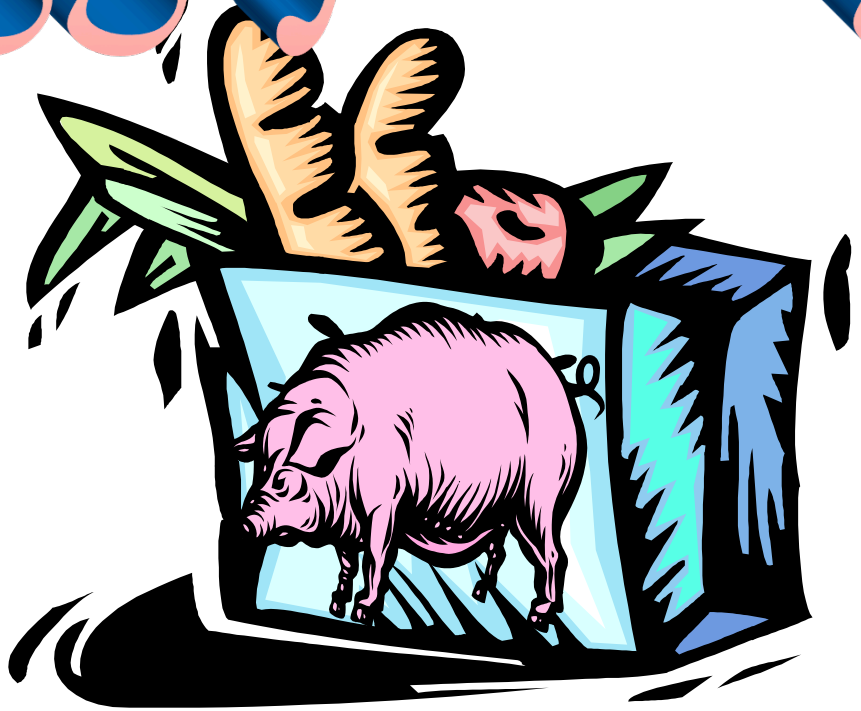
Slope

Systems of Equations

Finding equation given two points

Statistics

Puggly Wuggly



No sale—just our usual price:

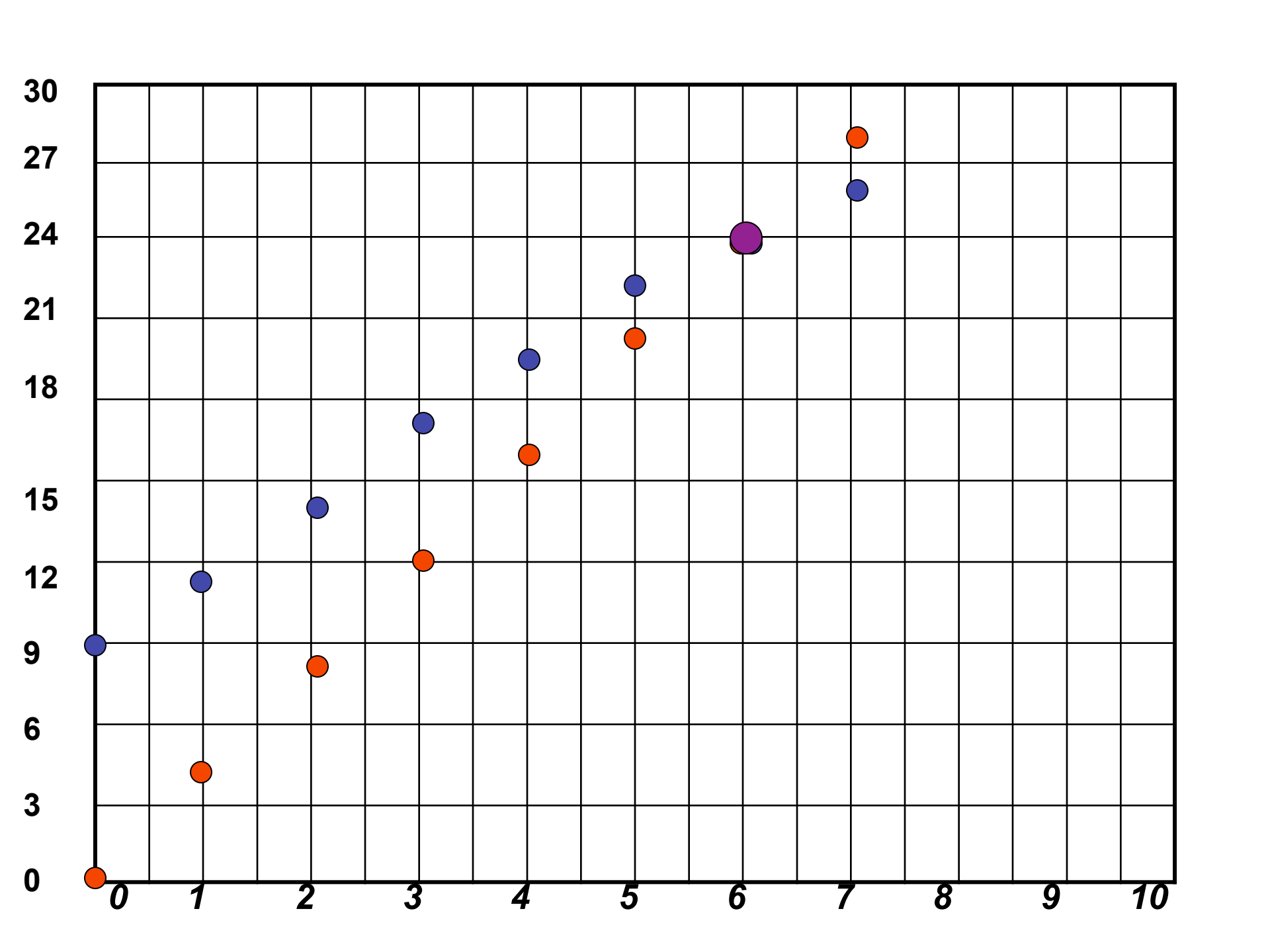
$2.50 per item

Fancy Foods

The best food at the worst prices!

All organic food

$4 per item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18
15
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0
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9
10

Puggly Wuggly vs. Fancy Foods script

Teacher 1—There's no way you will be able to shop cheaper than me with that ridiculous car of yours. What does it get– 7 miles to the gallon? That's unbelievable—The Puggly Wuggly is 10 miles a way. Round trip that's 20 miles. You will spend $9 on gas just driving back and forth to the Puggly Wuggly. What a waste of money. I'd rather stay here and walk next door to the Fancy Foods.

Teacher 2—you might be right, but I think I'll save money at the Puggly —I mean the prices are so much cheaper. I've got to win!

Teacher 1– are you there yet? Okay, I've got my first item. It cost me $4. What about you.

Teacher 2—well you were right, I did the math on it, and I am going to need $9 in gas. But my first item is only $2.50.

Puggly Wuggly vs. Fancy Foods script

Teacher 1—Hah—that means you've already spent 11.50! You are in trouble. I've got another item, and now I'm still only at $8.

Teacher 2—I see what you mean. I just picked up my second item, let's see that's 2 x 2.50→ $5 plus that darn $9 for gas…I'm up to $14.00.

Teacher 1—I'm loving Fancy Foods, you know they don't use any Bovine Growth Hormone in their milk here…I'm loving life, okay, my 3rd item gets me up to 4(3) = $12.

Teacher 2—well maybe I should give up. I'm still ahead of your cost. I'm at 7.50 plus that $9, so I'm at $16.50. What were you at again? $12? I'm cooked—no pun intended.

Teacher 1—don't feel too bad, you might as well keep shopping and get something done. I'm up to 4 items now, and that's $16.00.

Puggly Wuggly vs. Fancy Foods script

Teacher 2—Let's see, for me, 4 items cost, let's see what is it 4(2.50) that's $10 plus the dang $9.00—you know I'm really starting to resent that guzzling monster. That's 19.00 for me.

Teacher 1—I've got 5 items now, still cruising here at just $20.00. Life is good.

Teacher 2—Yeah, yeah, yeah. I've got 5 items too and I'm at 5(2.50) is 12.50 plus 9 is 21.50.

Teacher 1—Well I'm almost done, just going to pick up a few more items. I'm getting item number 6 here, some delicious garbanzo beans, for $4 more dollars. I'm at $24.

Teacher 2—alright, those beans sound good, and I guess I give up. Let me see I'll get these green beans for 2.50 more and that puts me at $15.00 plus $9, that's $24.

Teacher 1—what did you say? $24.00? That's impossible. That's what 6 items cost for me! How did that happen?

Puggly Wuggly vs. Fancy Foods script

Teacher 2—Gosh, I don't know. I mean, I teach algebra, but I don't know anything about grocery shopping.

Teacher 1—I'm nervous about this, but let's see what happens when we buy one more item. Okay, I'm going to be at 7 items times $4 that's $28.00 what about you?

Teacher 2—hmmm, 7(2.50)I'll use the distributive property to figure this out in my head—(7 times 2 dollars is 14 plus 7 times .50 is 3.50. so my groceries are 17.50, plus that Constant, Nagging $9 for gas. That's 26.50 for me. Hey—that's Cheaper than you! That's awesome…Let's keep shopping!

Teacher 1—You know, I don't like what's happening here, I think I'm going to check out. I should have quit while I was ahead! Enjoy your green beans.

Teacher 2—Enjoy your bovine-growth-hormone-free milk, and I'll see you back at school.

Differentiation

Sameation

Given two points, find the equation of the line…

$$\frac{Y_1 - Y_2}{X_1 - X_2} \qquad \frac{\triangle Y}{\triangle X}$$

How are your students Processing this?

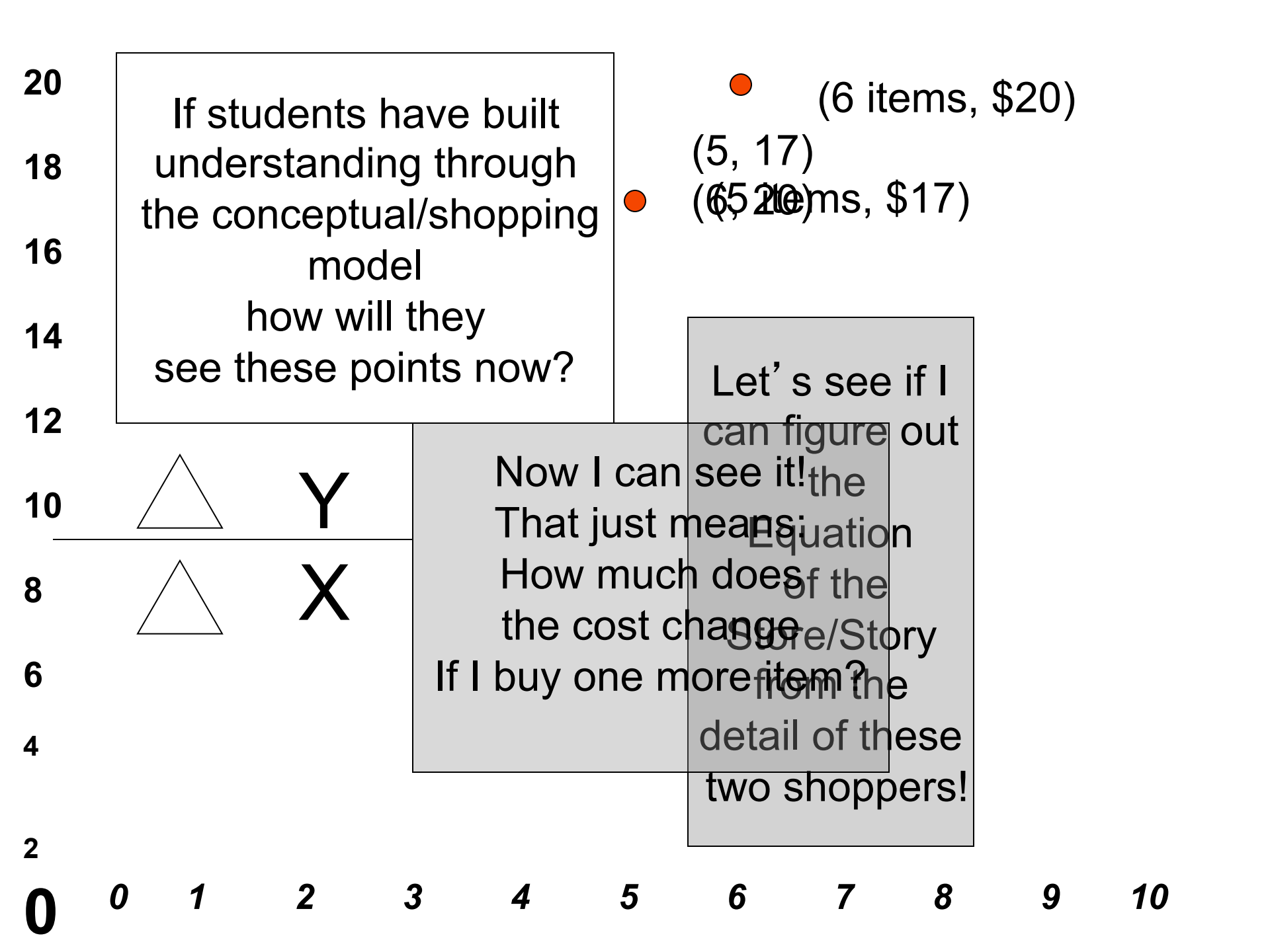
How can I get from here:

(5,17)
(6, 20)

to here:

$3x + 2 = y$

with MEANING?

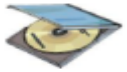

If students have built understanding through the conceptual/shopping model
how will they see these points now?
(6 items, $20)
(5, 17)
(6, 20)
(5, 17)
Now I can see it!
That just means:
How much does the cost change
If I buy one more item?
Let’ s see if I can figure out the Equation of the Store/Story from the detail of these two shoppers!
Y
X
20
18
16
14
12
10
8
6
4
2
0
0 1 2 3 4 5 6 7 8 9 10

Seeing **Structures**

Creating Equations

Reasoning with Equations

Verbal:



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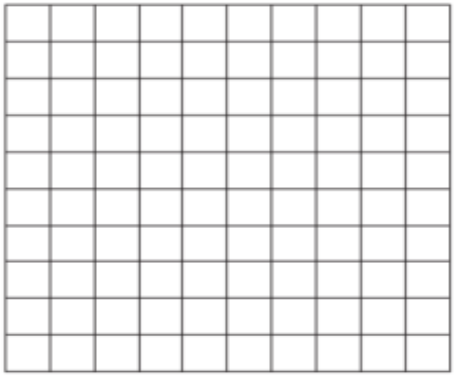
Algebraic

Table

_____(x)	Function: _____	_____ (y)

Graph

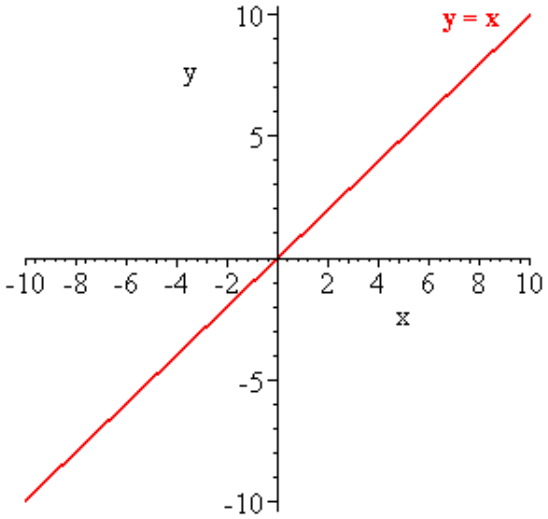


Wow!
I can't wait
to teach this way
with my students!

Conceptual Algebra

Notes and appendix

Classroom Implementation



Identity Function

$$y = x$$

Dollar Deals

Direct Variation

$$y = kx$$

Dollar Deals and Puggly Wuggly

Slope-Intercept Form

$$y = mx + b$$

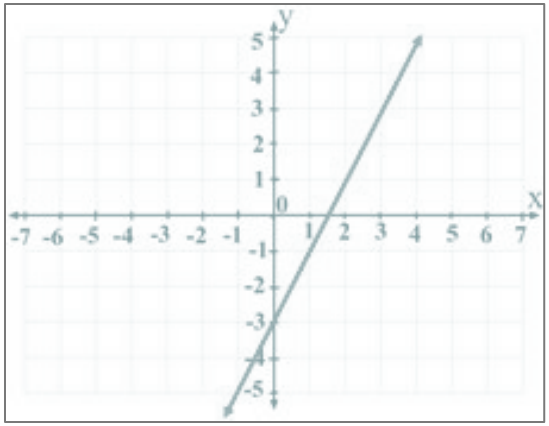
Feeding the DumDee

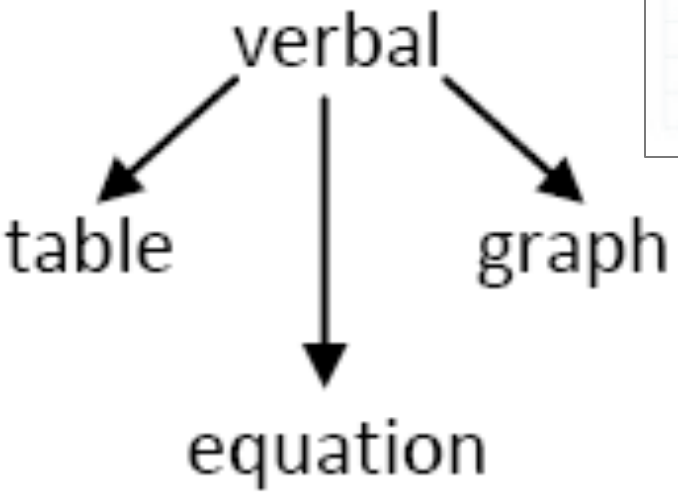
$$y = \begin{pmatrix} rate\ of \\ change \end{pmatrix} x + \begin{pmatrix} initial \\ amount \end{pmatrix}$$

Write the Equation Given Two Points

$$y = mx + b$$

x	y
–2	3
–1	1
0	–1
1	–3





Number of Items and Total Cost

Standard Form

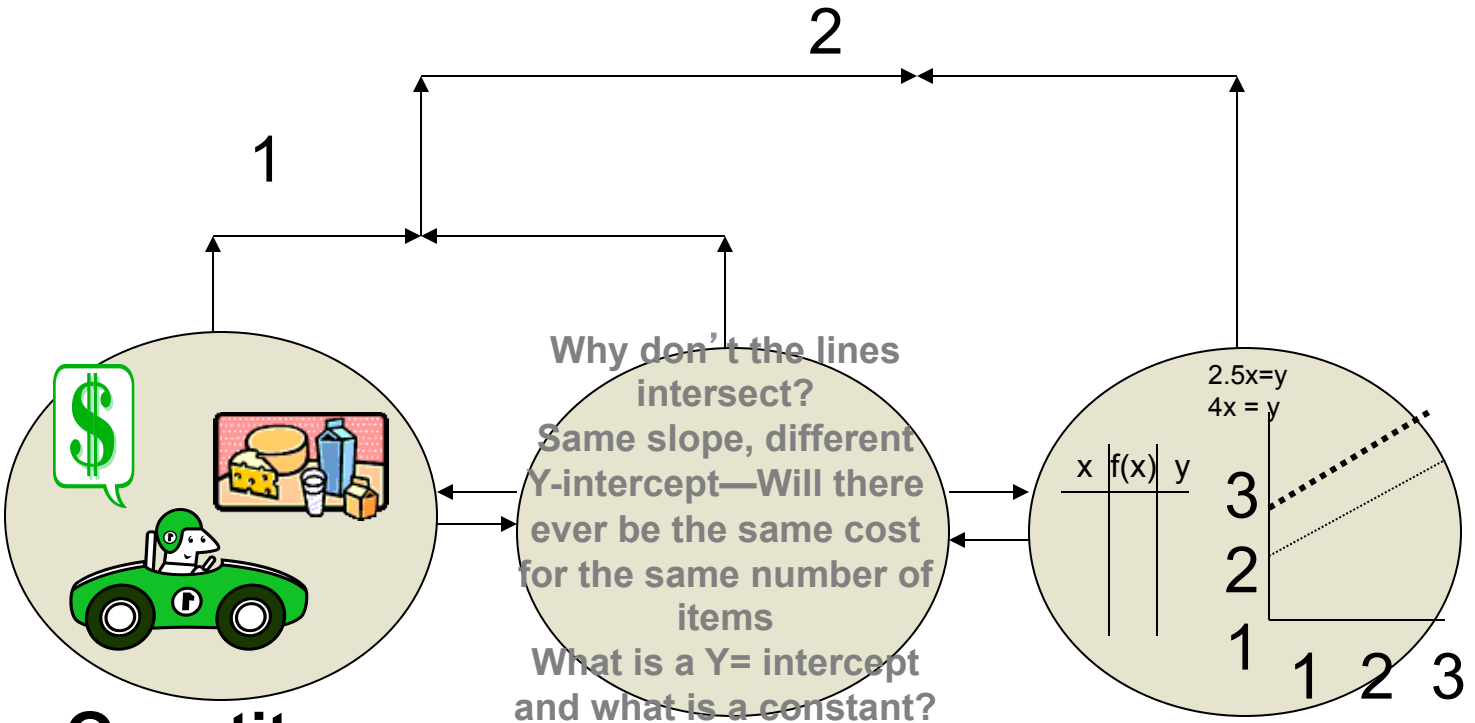
$$ax + by = c$$

Money

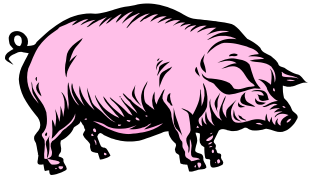
Mixtures - Crunch Berries

$$(Value)x + (Value)y = Total$$

Feeding the DumDee--Jenn V. Jermaine Learning Objectives



Mathematical Models



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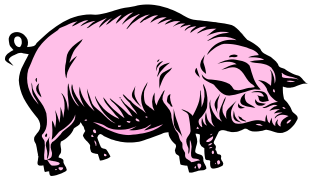

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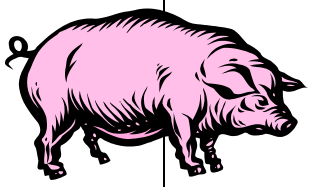
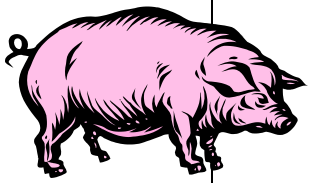
x Number of Items	Rule Math Model	y Cost
	+	

x Number of Items	Rule Math Model	y Cost
	+	

Two shoppers, One purpose

Jenn and Jermaine had a little argument about shopping. Jenn said that she could shop for things cheaper than Jermaine, because "guys don't know how to shop." Jermaine thought that was ridiculous and challenged her to a shopping match. "You're on" she said. "Great" said Jermaine, "Save your receipt and you need to include the cost of travel to get to the store." Jenn agreed to these conditions and they were ready to go.

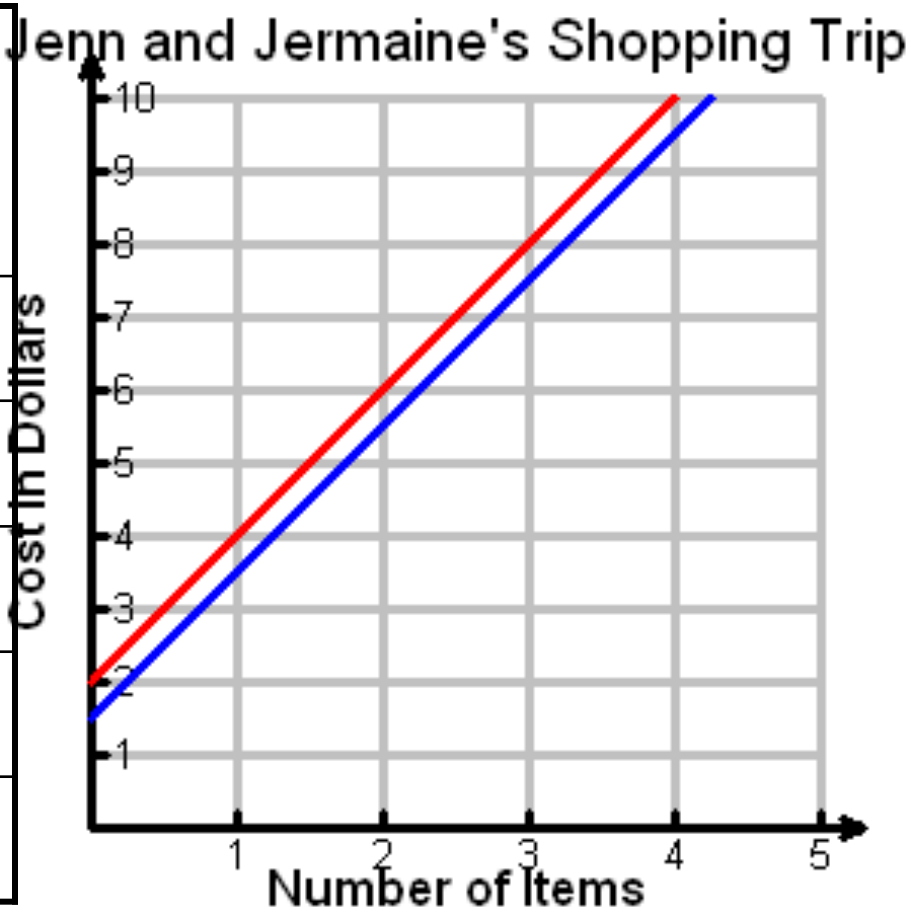
Jenn had an energy efficient hybrid car, so she felt confident that she would win, because Jermaine drove a truck. Puggly Wuggly was far away, but she thought the cheap prices at Puggly's $2/item bag day sale would be worth the drive. She spent $2.00 on gas for the whole trip.

Jermaine knew about the Puggly Bag Sale too. But he didn't drive his truck. He took the bus and it cost him $1.50.

Model the contest between the two with linear equations and then graph them. Answer the questions and be prepared to discuss with the class.

Jenn and Jermaine's Shopping Trip

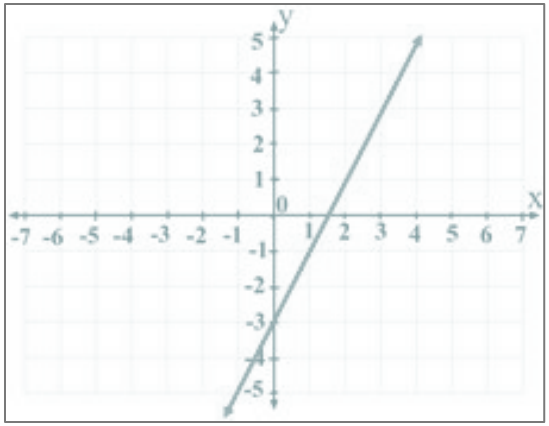
X Items	Y=2x + 1.5 Jermaine	Y=2x + 2 Jenn
0	$1.50	$2.00
1	$3.50	$4.00
2	$5.50	$6.00
3	$7.50	$8.00
4	$9.50	$10.00

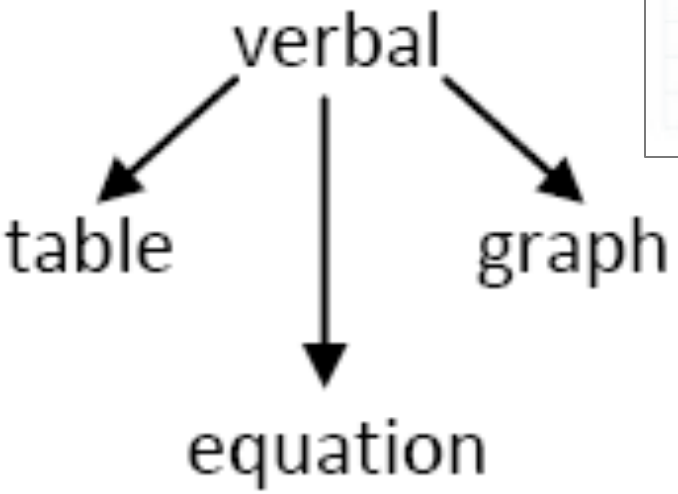


Write the Equation Given Two Points

$$y = mx + b$$

x	y
–2	3
–1	1
0	–1
1	–3





Number of Items and Total Cost

Example student dialogue:

That means that someone bought 5 items and it cost them 17 dollars and someone else bought 6 items and it cost them 20 dollars.

Great! But does this give me the information I need To create an equation for the store?

No.

Why not?

No cost per item/no membership fee information

Okay, so…

Let’s make sense of what we know…

Start with examples that are only one item apart and then build from there!

- The arithmetic won't get in the way and the CONCEPTS can be understood and therefore discussed!

Reflection on Two Themes

Three minutes - Jot some notes

Five minutes - Discuss with small group

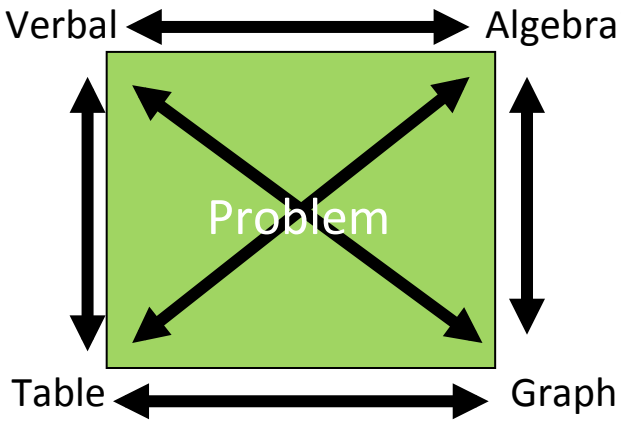
Central Conceptual Structures

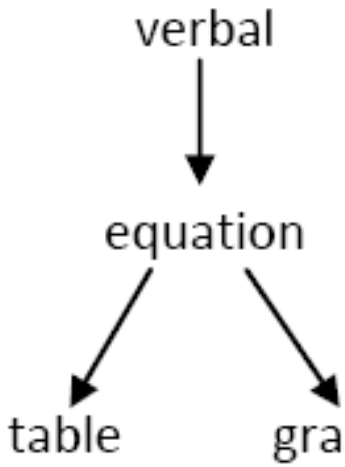
Re-consider how you Task Analyze when introducing concept and providing student support

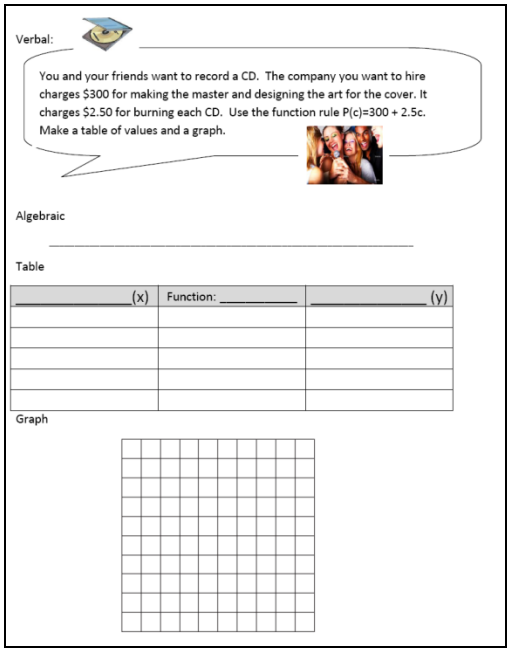
Appendix:
The Progression for Teaching Linear Functions, DumDee Script, and Jenn and Jermaine

Four Forms

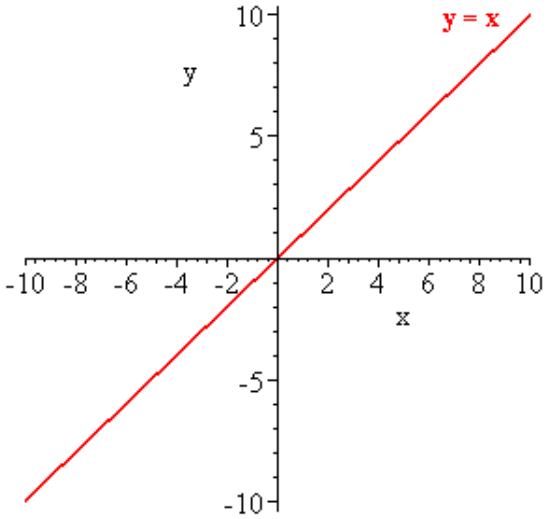
$$y = mx + b$$







Make connections between the four forms – how do you determine the slope from each form? How do you determine the y-intercept from each form?



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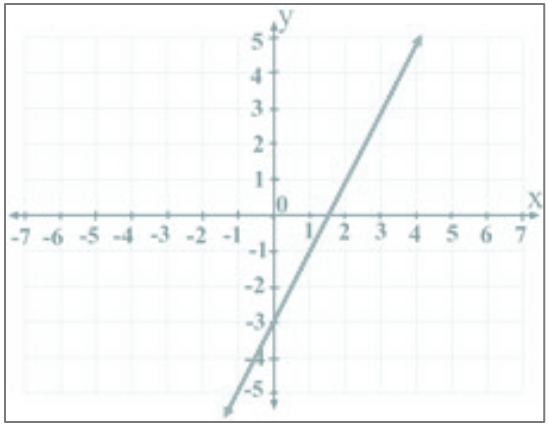

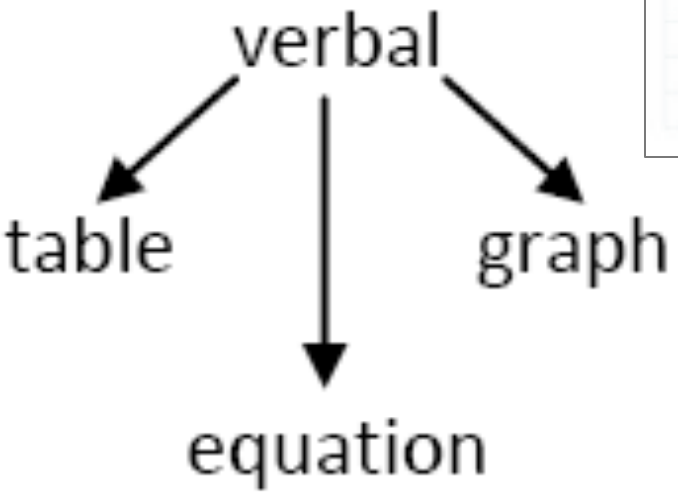

Number of Items and Total Cost

Standard Form

$$ax + by = c$$

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Mixtures - Crunch Berries

$$(Value)x + (Value)y = Total$$

Concrete Reality

8 - 5 = 8

7 - 4 = 7

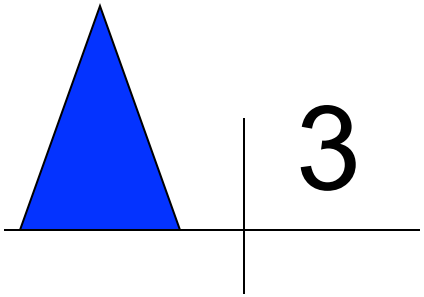
Tracy's construct is legitimate

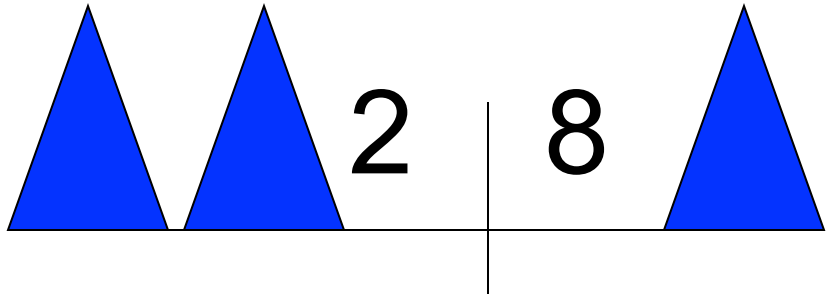
Eat my Steak

I still have my Potato!

http://www.hvmag.com/Hudson-Valley-Magazine/March-2009/Steak-Houses/Steak_Schlesingers9673.jpg

Hands on Equations Borenson.com





Lesson #1

What's the 'bug' in our kids' systems regarding Equality?

"Same As"