Reaching All Students:
Math and the Common Core

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International Research

TIMSS

from Improving Mathematics Instruction (Ed Leadership 2/2004)

- 1995 Video Study
  - Japan, Germany, US
  - Teaching Style Implicated
- 1999 Video Study
  - US, Japan, Netherlands, Hong Kong, Australia, Czech Rep.
  - Implementation Implicated

Style vs.. Implementation

- High Achieving use a variety of styles to teach
- High Achieving implement connections problems as connections problems
- U.S. implements connection problems as a set of procedures

Defining Issue in Implementation

…is the teacher's <u>own</u> understanding of Mathematics.

--Liping Ma

Exponents and Geometry

What is 4^2 ?

Why is it 4 x 4 when it looks like 4 x 2?

It means ‘make a square out of your 4 unit side’

Exponents and Geometry

What is 4^2 ?

--4 units--

1

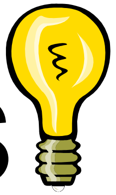
1

1

1

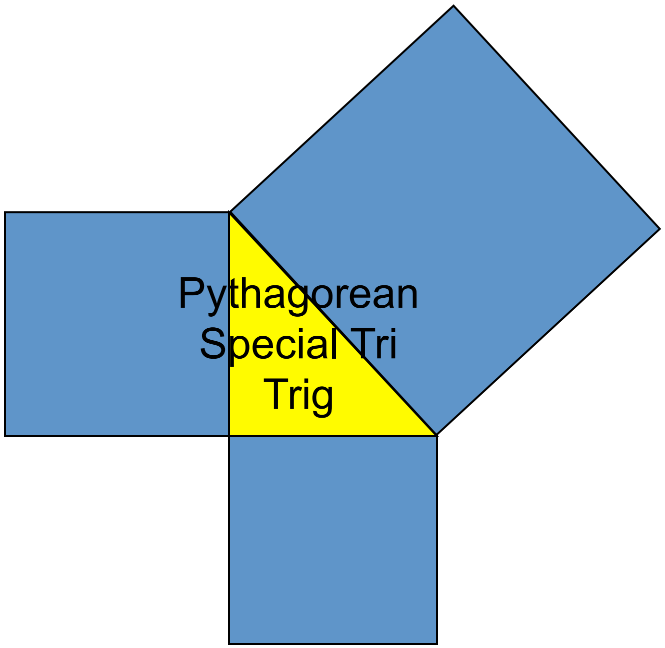
You' d get how many little 1 by 1 inch squares?

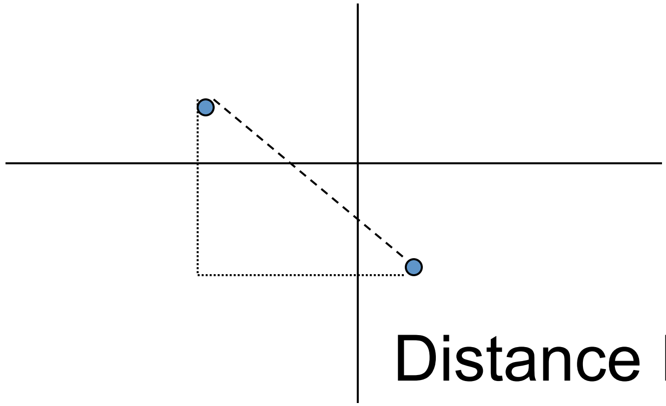
Exponents and CONNECTIONS



Square Roots!
$\sqrt{16} = 4$
The length of one side!

Geometry and Measurement

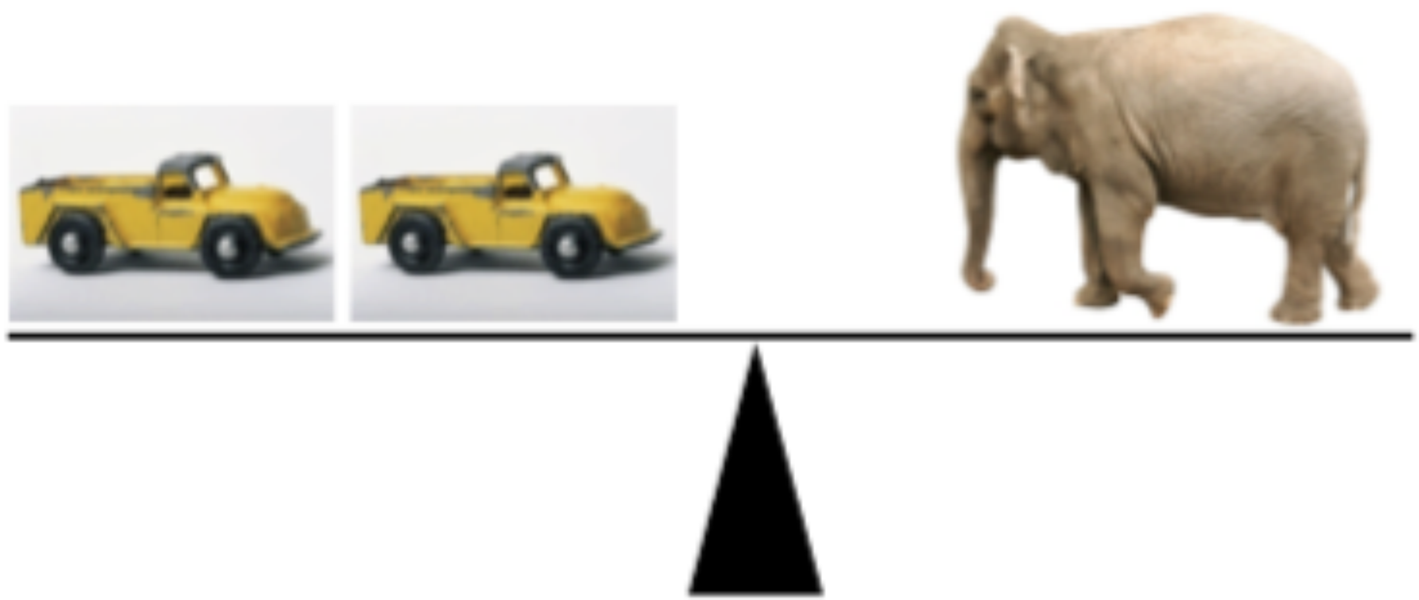




Distance Formula

$$1 = 1^3$$

‘Same as’



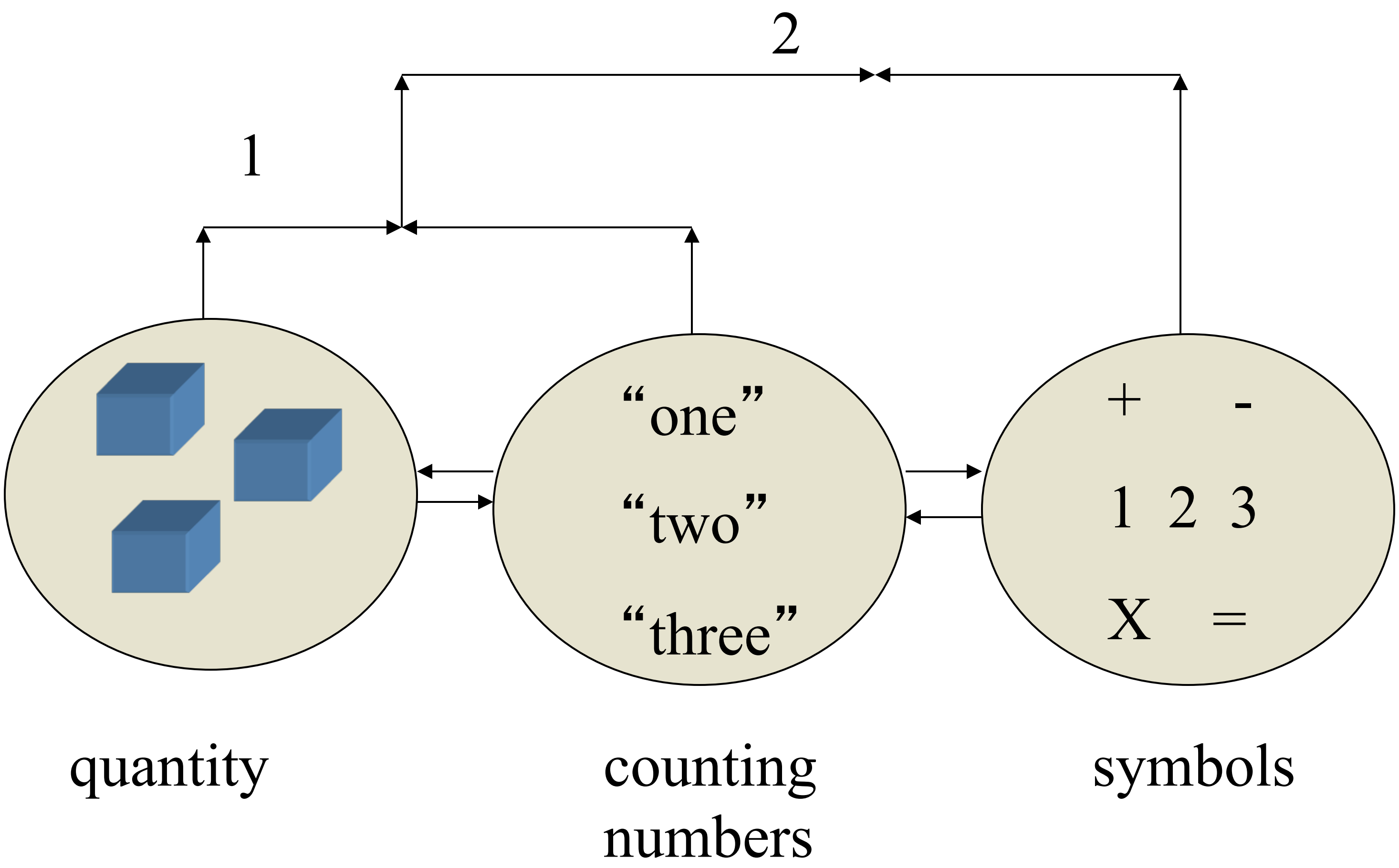
This is the ESSENCE of shifting to the Common Core Standards:

It's a cultural shift that demands changes at the level of

Visual Structures and Language.

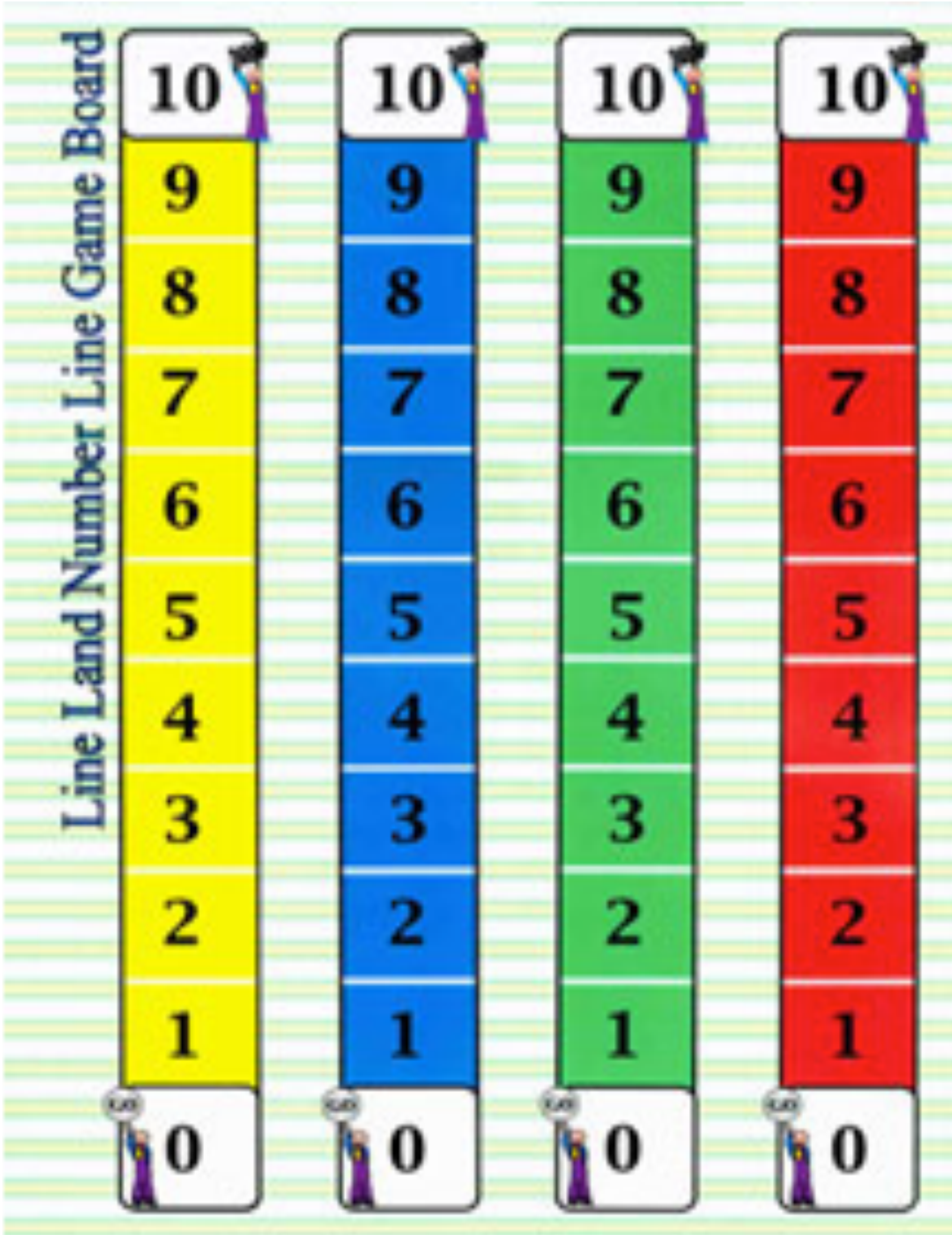
Key Mathematical Structures

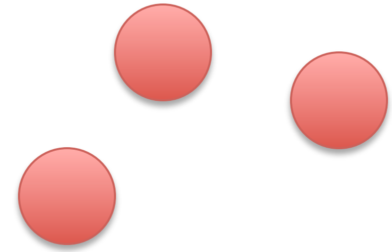
Case, Griffin, Okamoto

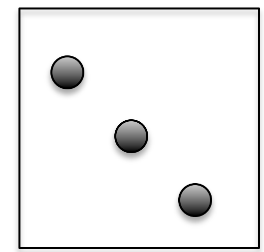


Key Mathematical Structure #1 The number line:

Connecting Quantity to Magnitude







From Sharon Griffin
Number Worlds

Common Core Standard and Cluster

Describe and compare measurable attributes.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **length, weight, heavy, long, more of, less of, longer, taller, shorter.**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.	Students describe measurable attributes of objects, such as length, weight, size, and color. For example, a student may describe a shoe with one attribute, "Look! My shoe is blue, too!", or more than one attribute, "This shoe is heavy! It's also really long."
K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*	Direct comparisons are made when objects are put next to each other, such as two children, two books, two pencils. For example, a student may line up two blocks and say, "The blue block is a lot longer than the white one." Students are not comparing objects that cannot be moved and lined up next to each other.  Similar to the development of the understanding that keeping track is important to obtain an accurate count, kindergarten students need ample experiences with comparing objects in order to discover the importance of lining up the ends of objects in order to have an accurate measurement. As this concept develops, children move from the idea that "Sometimes this block is longer than this one and sometimes it's shorter (depending on how I lay them side by side) and that's okay." to the understanding that "This block is always longer than this block (with each end lined up appropriately)." Since this understanding requires conservation of length, a developmental milestone for young children, kindergarteners need multiple experiences measuring a variety of items and discussing findings with one another.  "Sometimes this block is longer and sometimes it's shorter."  "The dark block is always longer than this block"

Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.*

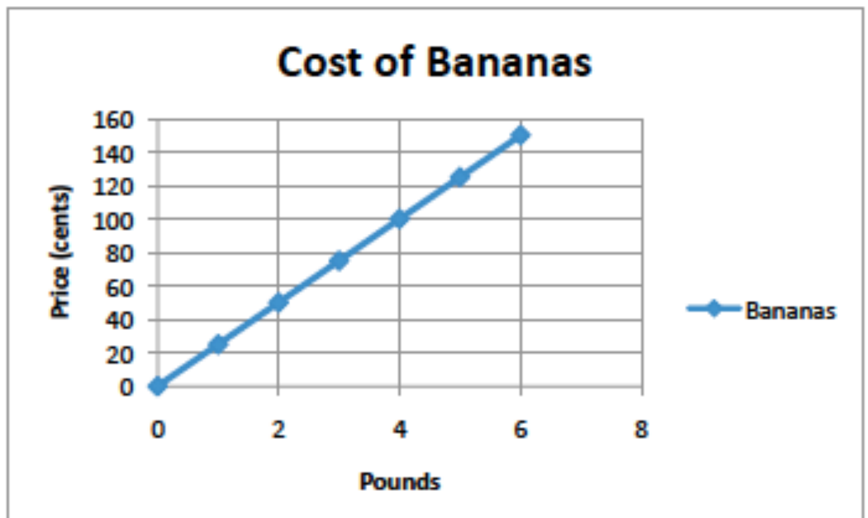
c. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.

Solution:
Students can examine the numbers to determine that the price is the number of books multiplied by 3, except for 7 books. The row with seven books for $18 is not proportional to the other amounts in the table; therefore, the table does ***not*** represent a proportional relationship.

Students graph relationships to determine if two quantities are in a proportional relationship and to interpret the ordered pairs. If the amounts from the table above are graphed (number of books, price), the pairs (1, 3), (3, 9), and (4, 12) will form a straight line through the origin (0 books, 0 dollars), indicating that these pairs are in a proportional relationship. The ordered pair (4, 12) means that 4 books cost $12. However, the ordered pair (7, 18) would not be on the line, indicating that it is not proportional to the other pairs.

The ordered pair (1, 3) indicates that 1 book is $3, which is the unit rate. The y-coordinate when $x = 1$ will be the unit rate. The constant of proportionality is the unit rate. Students identify this amount from tables (see example above), graphs, equations and verbal descriptions of proportional relationships.

<u>Example 2:</u>
The graph below represents the price of the bananas at one store. What is the constant of proportionality?



Solution:
From the graph, it can be determined that 4 pounds of bananas is $1.00; therefore, 1 pound of bananas is $0.25, which is the constant of proportionality for the graph. Note: Any point on the line will yield this constant of proportionality.

Key
Mathematical Structure #2a
Expanding to a
Circuit Number Line:

Understanding the Recursive nature/pattern of Base-Ten Additively and Proportionally

Number and Operations in Base Ten — K.NBT

Common Core Standard and Cluster

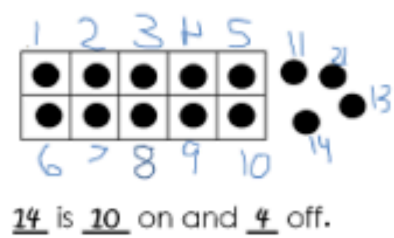
Work with numbers 11–19 to gain foundations for place value.
Rather than unitizing a ten (recognizing that a set of 10 objects is a unit called a "ten"), which is a standard for First Grade (1.NBT.1a), kindergarteners keep each count as a single unit as they explore a set of 10 objects and leftovers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: number words (**one, two… thirteen, fourteen, … nineteen**), **leftovers**

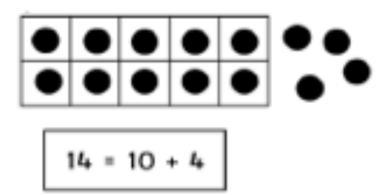
Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8)*; understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. * Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.	Students explore numbers 11-19 using representations, such as manipulatives or drawings. Keeping each count as a single unit, kindergarteners use 10 objects to represent "10" rather than creating a unit called a ten (unitizing) as indicated in the First Grade CCSS standard 1.NBT.1a: 10 can be thought of as a bundle of ten ones — called a "ten."

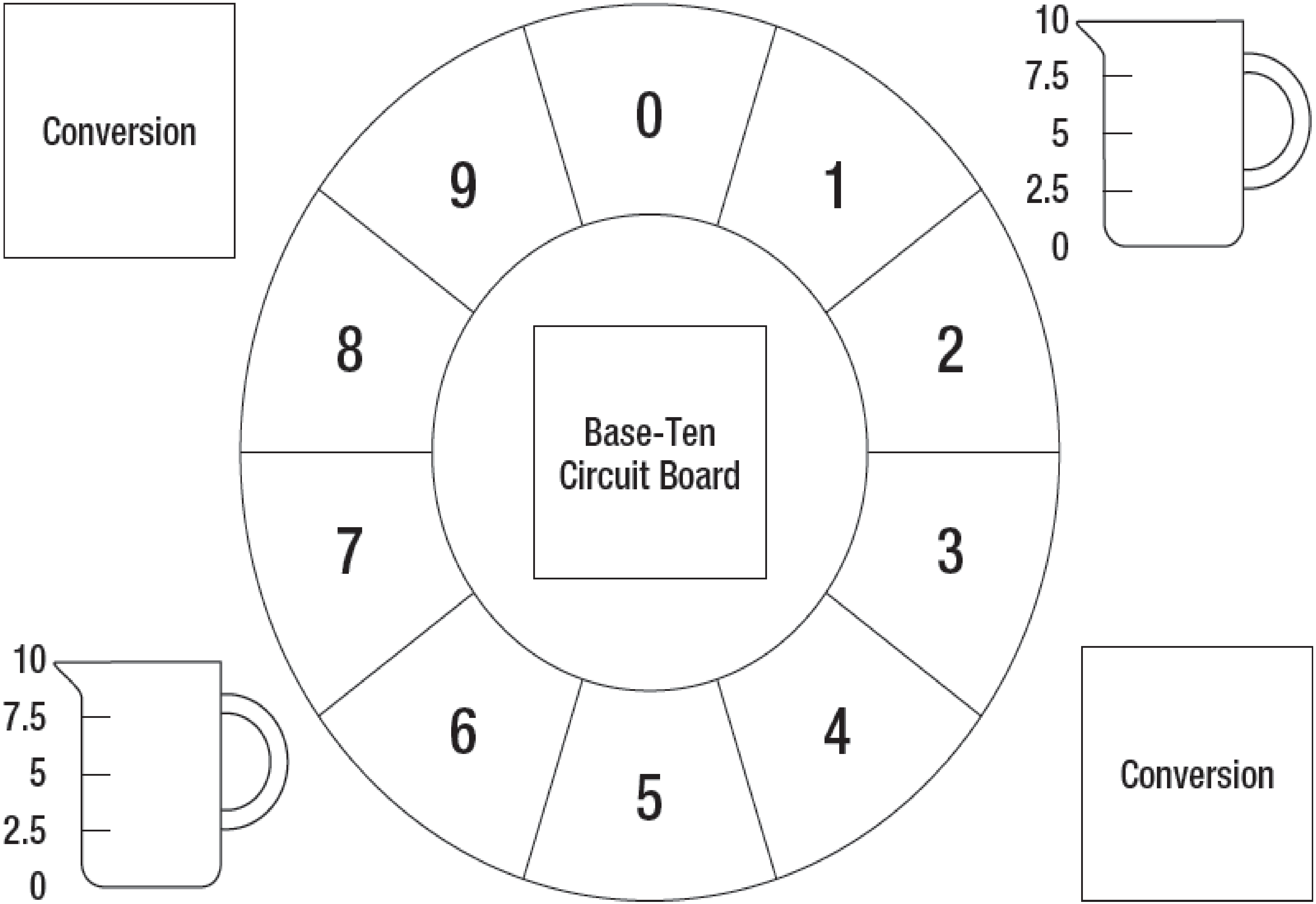
Example:
Teacher: "I have some chips here. Do you think they will fit on our ten frame? Why? Why Not?"
Students: Share thoughts with one another.
Teacher: "Use your ten frame to investigate."
Students: "Look. There's too many to fit on the ten frame. Only ten chips will fit on it."
Teacher: "So you have some leftovers?"
Students: "Yes. I'll put them over here next to the ten frame."
Teacher: "So, how many do you have in all?"
Student A: "One, two, three, four, five… ten, eleven, twelve, thirteen, fourteen. I have fourteen. Ten fit on and four didn't."
Student B: Pointing to the ten frame, "See them- that's 10… 11, 12, 13, 14. There's fourteen."
Teacher: Use your recording sheet (or number sentence cards) to show what you found out.

Student Recording Sheets Example:

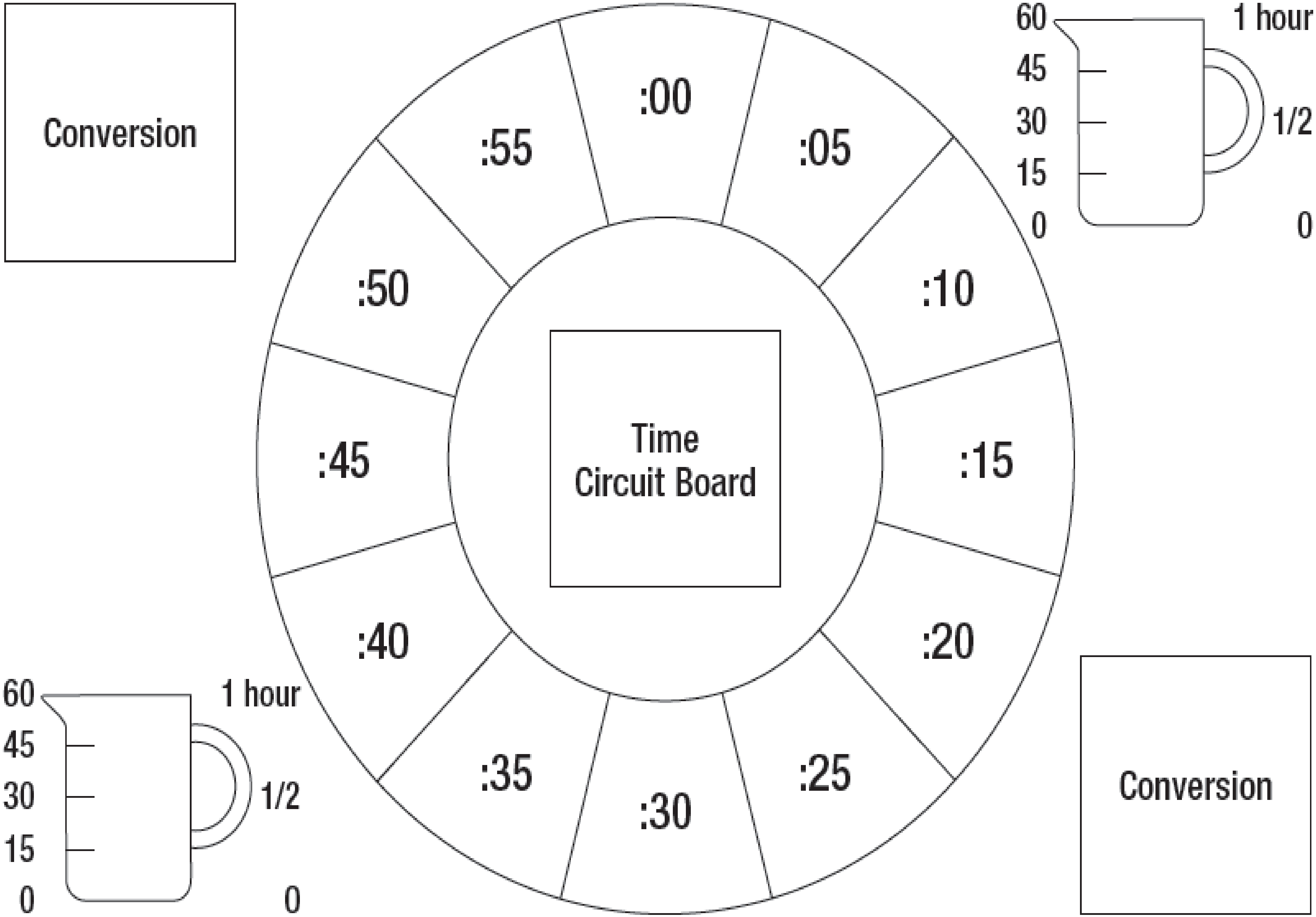


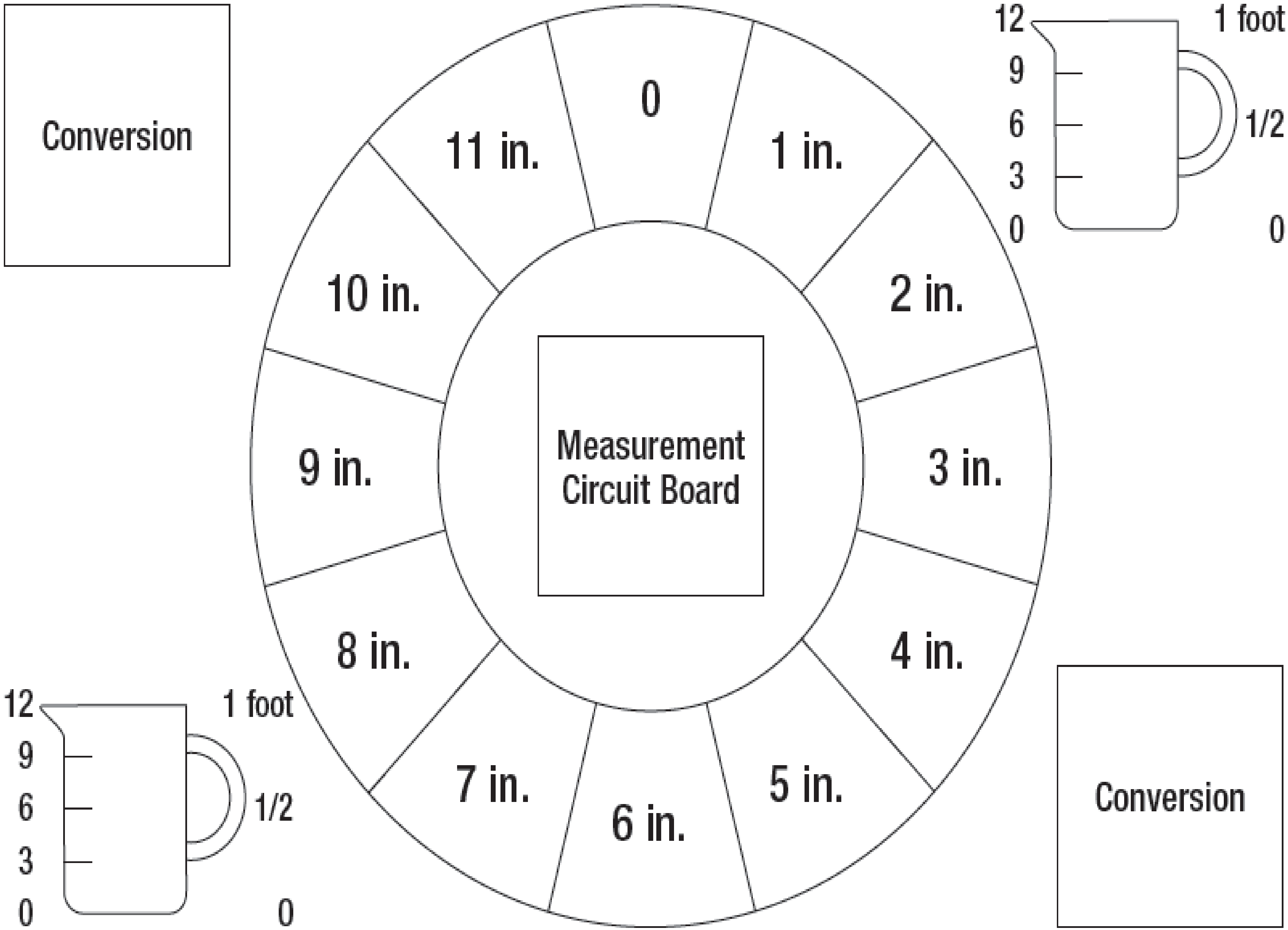
All	On	Off
14	10	4





Adapted from Griffin, Number Worlds, SRA


Conversion
:00
:05
:10
:15
:20
:25
:30
:35
:40
:45
:50
:55
Time
Circuit Board
60
45
30
15
0
1 hour
1/2
0
Conversion
60
45
30
15
0
1 hour
1/2
0



♥Valerie Faulkner Introductory Math, Walch Publications

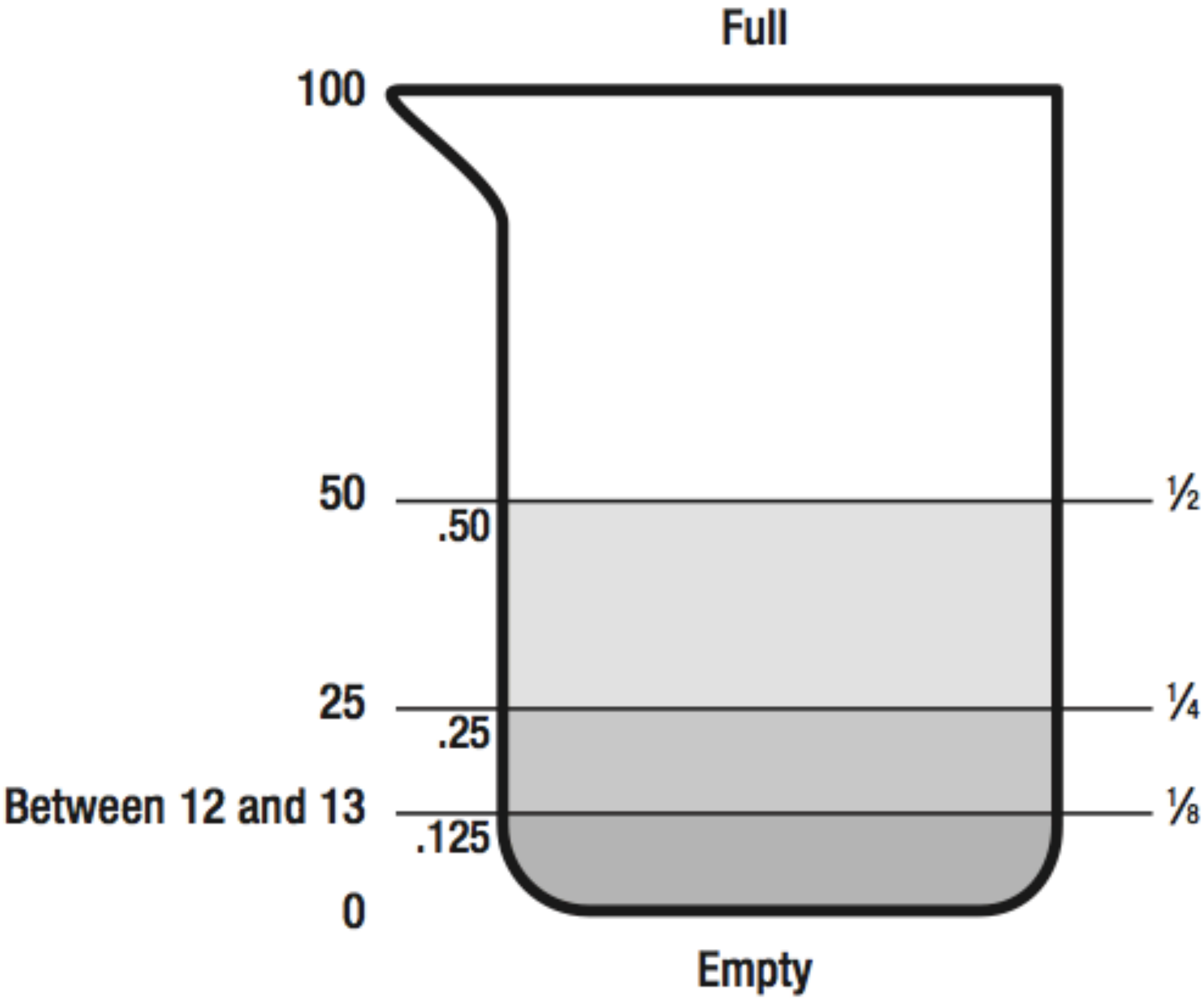
1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*	First Graders use non-standard objects to measure objects which help students focus on the attribute being measured. A non-standard object also lends itself to future discussions regarding the need for a standard unit. First Grade students use multiple copies of one object to measure the length larger object. Through numerous experiences and careful questioning by the teacher, students will recognize the importance of careful measuring so that there are not any gaps or overlaps in order to get an accurate measurement. This concept is a foundational building block for the concept of area in 3^{rd} Grade. Example: **How long is the pencil, using paper clips to measure?** **Student**: I carefully placed paper clips end to end. The pencil is 5 paper clips long. I thought it would take about 6 paperclips. 

Key
Mathematical Structure #3
Rational Numbers live on the number line:

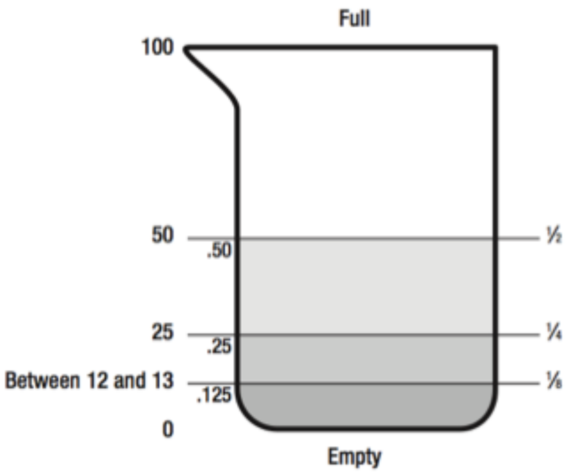
Connecting whole number understandings to rational numeration systems

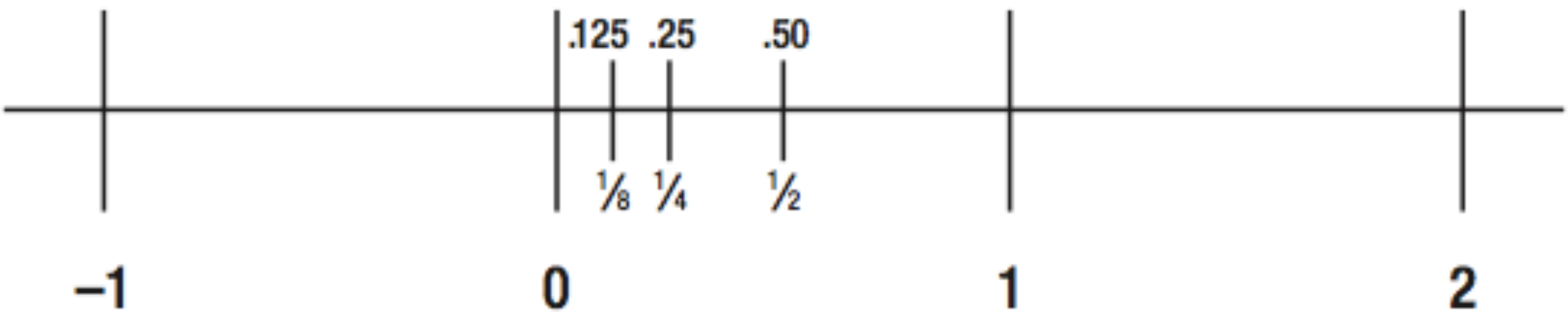
Fractions and Percents I

Draw the following on the board and model this as the lesson proceeds.

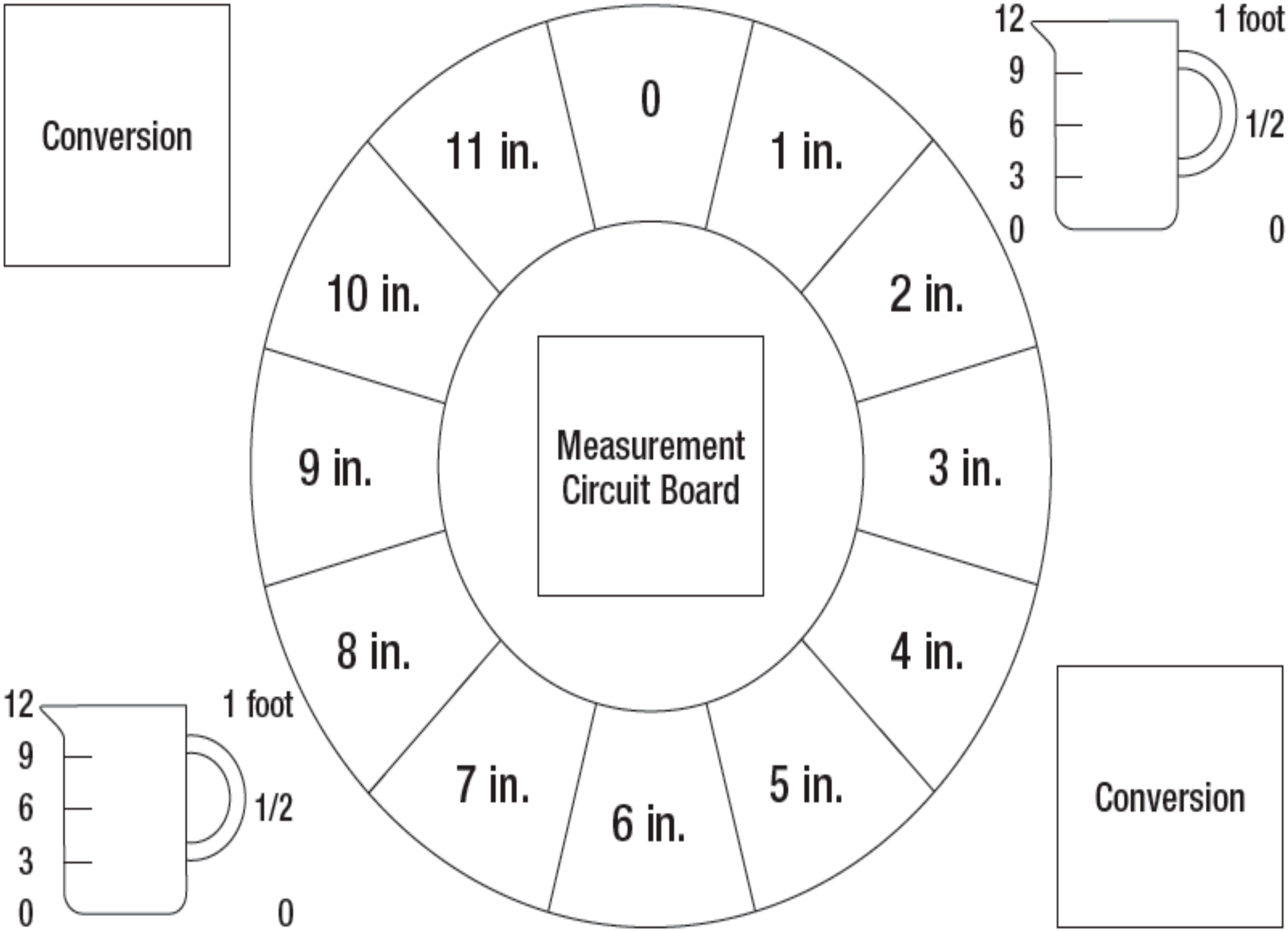


Faulkner,Dupree Walch Educational, 2010 adapted from Case, 2001





Faulkner,Dupree Walch Educational, 2010 adapted from Case, 2001



♥Valerie Faulkner Introductory Math, Walch Publications

Common Core Cluster	
Work with time and money.	
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **clocks, hand, hour hand, minute hand, hour, minute, a.m., p.m., o'clock,** *multiples of 5* (e.g., **five, ten, fifteen,** etc.), **analog clock, digital clock, quarter 'til, quarter after, half past, quarter hour, half hour, thirty minutes before, 30 minutes after, 30 minutes until, 30 minutes past, quarter, dime, nickel, dollar, cent(s), $, ¢, heads, tails**	

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
2.MD.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	Second Grade students extend their work with telling time to the hour and half-hour in First Grade in order to tell (orally and in writing) the time indicated on both analog and digital clocks to the nearest five minutes. Teachers help students make connections between skip counting by 5s (2.NBT.2) and telling time to the nearest five minutes on an analog clock. Students also indicate if the time is in the morning (a.m.) or in the afternoon/evening (p.m) as they record the time. Learning to tell time is challenging for children. In order to read an analog clock, they must be able to read a dial-type instrument. Furthermore, they must realize that the hour hand indicates broad, approximate time while the minute hand indicates the minutes in between each hour. As students experience clocks with only hour hands, they begin to realize that when the time is two o'clock, two-fifteen, or two forty-five, the hour hand looks different- but is still considered "two". Discussing time as "about 2 o'clock", "a little past 2 o'clock", and "almost 3 o'clock" helps build vocabulary to use when introducing time to the nearest 5 minutes.  All of these clocks indicte the hour of "two", although they look slightly different. This is an important idea for students as they learn to tell time.

Procedures versus Connections

- High Achieving implement connections problems as connections problems
- U.S. implements connection problems as a set of procedures

K-3

15 - 8

How do you teach facts within 20?

4-12

1 3/8 – 5/8

How do you teach problems such as the above?

Is it possible that these two situations are, essentially, the exact same problem?

15 - 8

1 3/8 – 5/8

Key
Mathematical Structure #2b

Unit Size, Unit Size, Unit Size

Unit Size

3 ones and 2 ones

3 tens and 2 tens

3 tens and 2 ones

3/6   and 2/6

3/6 and 2/5

3X and 2X

3Y and 2Y

3X and 2Y

0 10ones and 5ones

15

\- 8

0 8/8 and 3/8

1 3/8

\- 5/8

Common Core Cluster
Work with time and money.
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **clocks, hand, hour hand, minute hand, hour, minute, a.m., p.m., o'clock,** *multiples of 5* (e.g., **five, ten, fifteen, etc.**), **analog clock, digital clock, quarter 'til, quarter after, half past, quarter hour, half hour, thirty minutes before, 30 minutes after, 30 minutes until, 30 minutes past, quarter, dime, nickel, dollar, cent(s), $, ¢, heads, tails**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
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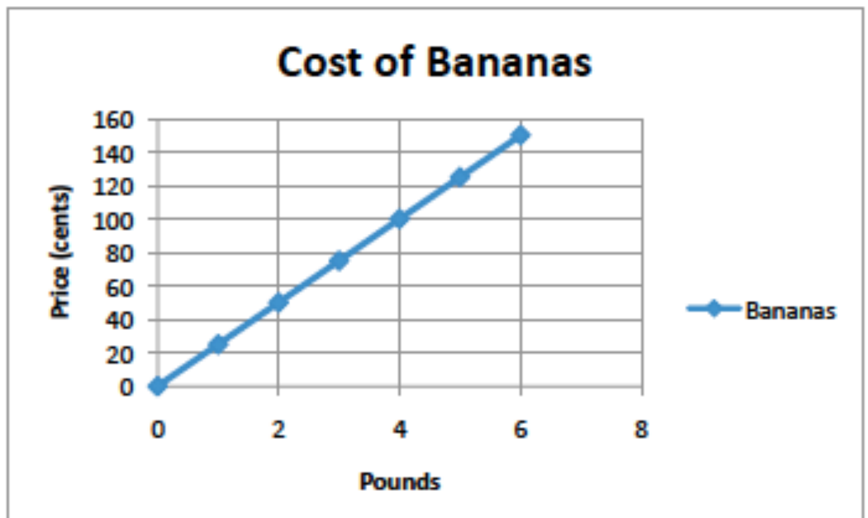
Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$. c. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.	*Solution:* Students can examine the numbers to determine that the price is the number of books multiplied by 3, except for 7 books. The row with seven books for $18 is not proportional to the other amounts in the table; therefore, the table does ***not*** represent a proportional relationship. Students graph relationships to determine if two quantities are in a proportional relationship and to interpret the ordered pairs. If the amounts from the table above are graphed (number of books, price), the pairs (1, 3), (3, 9), and (4, 12) will form a straight line through the origin (0 books, 0 dollars), indicating that these pairs are in a proportional relationship. The ordered pair (4, 12) means that 4 books cost $12. However, the ordered pair (7, 18) would not be on the line, indicating that it is not proportional to the other pairs. The ordered pair (1, 3) indicates that 1 book is $3, which is the unit rate. The y-coordinate when $x = 1$ will be the unit rate. The constant of proportionality is the unit rate. Students identify this amount from tables (see example above), graphs, equations and verbal descriptions of proportional relationships. <u>Example 2:</u> The graph below represents the price of the bananas at one store. What is the constant of proportionality?  *Solution:* From the graph, it can be determined that 4 pounds of bananas is $1.00; therefore, 1 pound of bananas is $0.25, which is the constant of proportionality for the graph. Note: Any point on the line will yield this constant of proportionality.

Precision and Structures

Making connections through the language you use

Decomposing a higher unit value into a lower unit…

using unit size to change the form of the number

Common Core Cluster

Use place value understanding and properties of operations to perform multi-digit arithmetic.[1]

[1] A range of algorithms may be used.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, round, addition, add, addend, sum, subtraction, subtract, difference, strategies, (properties)-rules about how numbers work**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.	This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.
3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. [1] A range of algorithms may be used.	This standard refers to fluently, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property). The word algorithm refers to a procedure or a series of steps. There are other algorithms other than the standard algorithm. Third grade students should have experiences beyond the standard algorithm. A variety of algorithms will be assessed on EOG. Problems should include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties. Students explain their thinking and show their work by using strategies and algorithms, and verify that their answer is reasonable. Example: There are 178 fourth graders and 225 fifth graders on the playground. What is the total number of students on the playground? **Student 1**: 100 + 200 = 300; 70 + 20 = 90; 8 + 5 = 13; 300 + 90 + 13 = 403 students **Student 2**: I added 2 to 178 to get 180. I added 220 to get 400. I added the 3 left over to get 403. **Student 3**: I know the 75 plus 25 equals 100. I then added 1 hundred from 178 and 2 hundreds from 275. I had a total of 4 hundreds and I had 3 more left to add. So I have 4 hundreds plus 3 more which is 403.

- 4th grade

Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

Example:

$1\frac{1}{4} - \frac{3}{4} = \square$ $\quad$ $4/4 + \frac{1}{4} = 5/4$ $\quad$ $5/4 - \frac{3}{4} = 2/4$ or $\frac{1}{2}$

Example of word problem:

Mary and Lacey decide to share a pizza. Mary ate 3/6 and Lacey ate 2/6 of the pizza. How much of the pizza did the girls eat together?

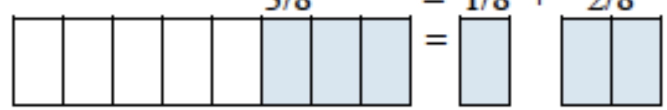
Possible solution: The amount of pizza Mary ate can be thought of a 3/6 or 1/6 and 1/6 and 1/6. The amount of pizza Lacey ate can be thought of a 1/6 and 1/6. The total amount of pizza they ate is 1/6 + 1/6 + 1/6 + 1/6 + 1/6 or 5/6 of the whole pizza.

Students should justify their breaking apart (decomposing) of fractions using visual fraction models. The concept of turning mixed numbers into improper fractions needs to be emphasized using visual fraction models.

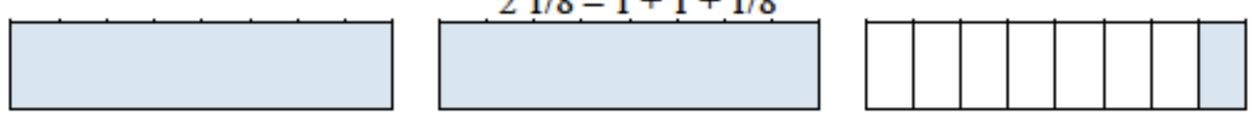
Example:

3/8 = 1/8 + 1/8 + 1/8

3/8 = 1/8 + 2/8

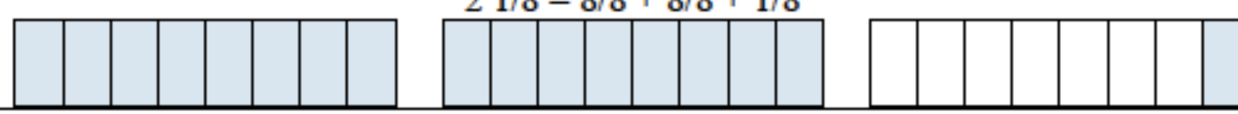


2 1/8 = 1 + 1 + 1/8



or

2 1/8 = 8/8 + 8/8 + 1/8



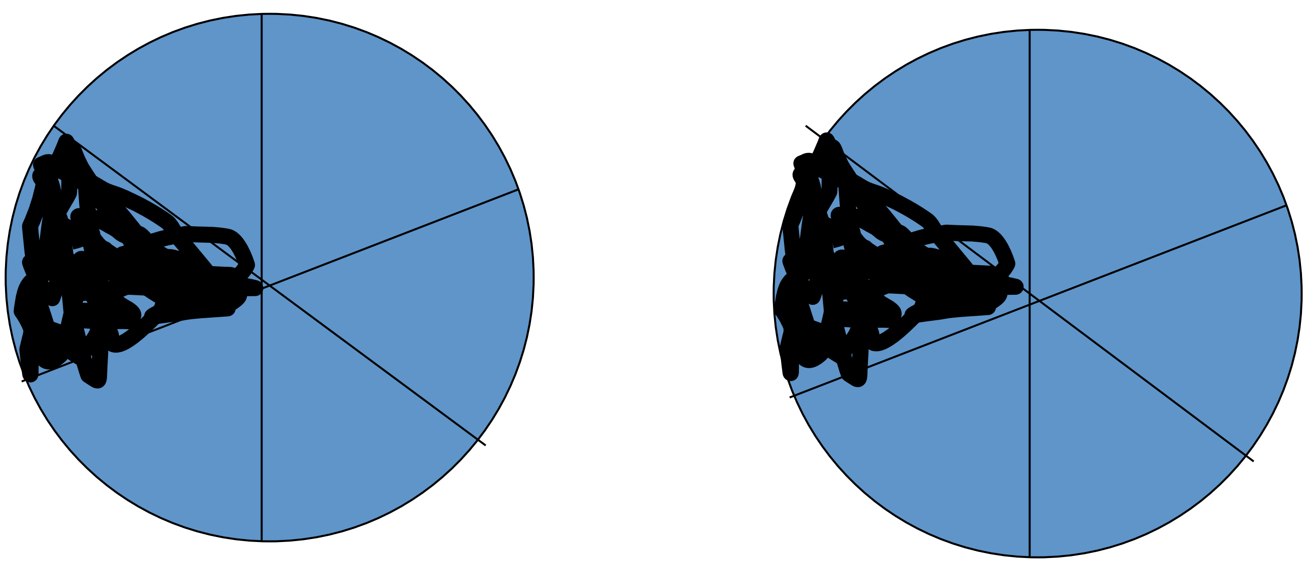
Manipulatives and "Magical Hopes"

Deborah Ball

How do you explain this to a student?

1/6 + 1/6 = 2/6

1/6 + 1/6 = 2/12?

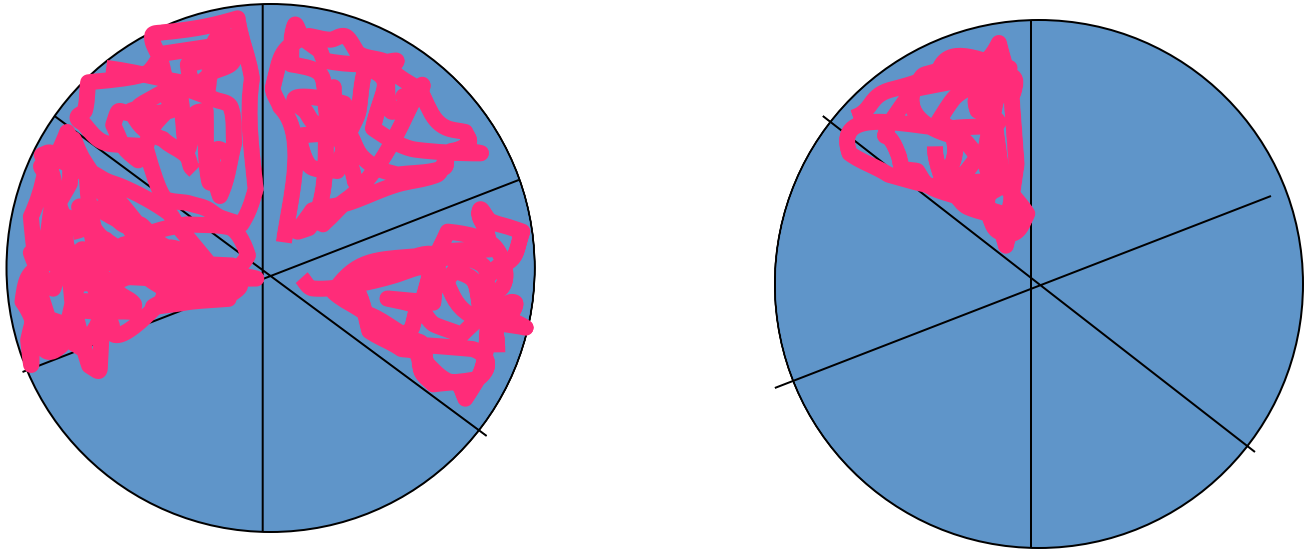


Game from SRA Real Math

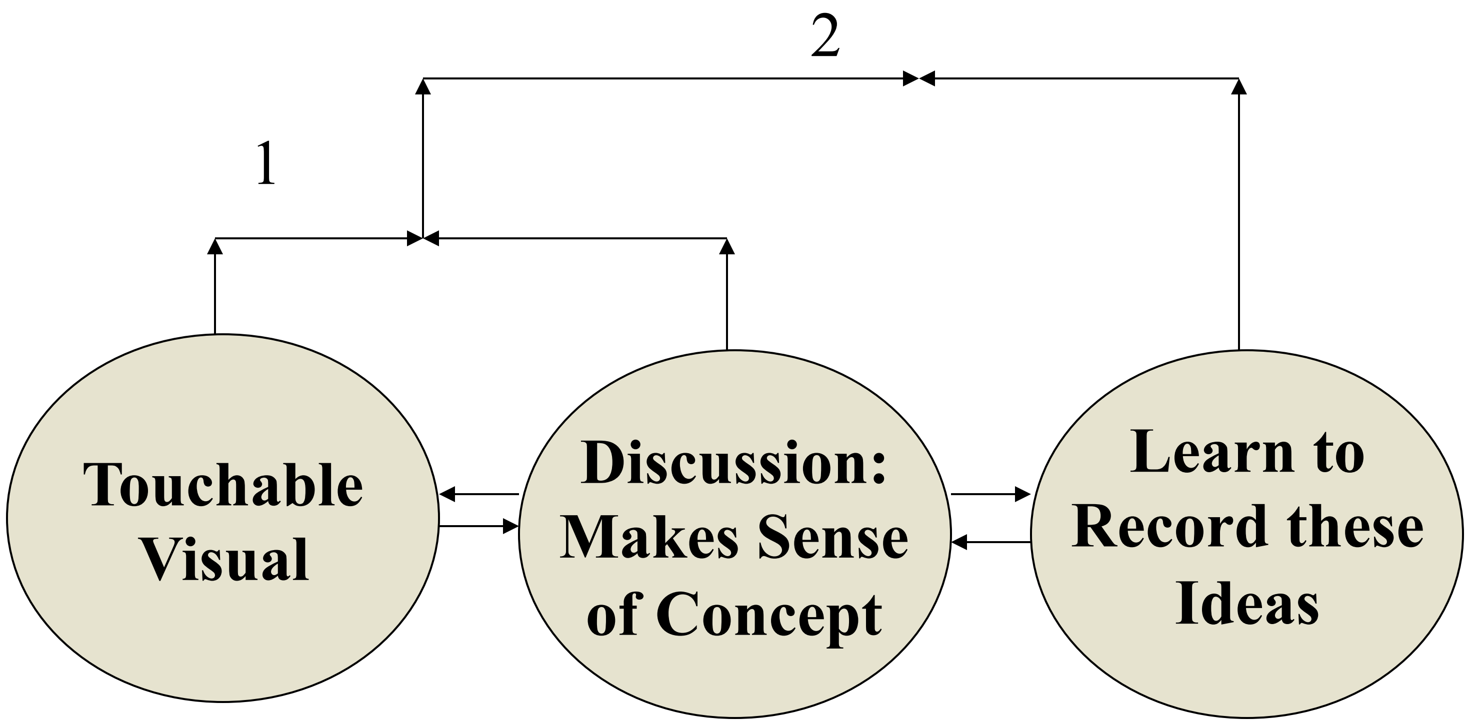
I roll 5/6

What should I do with my 5/6ths?

4/6 + 1/6 = 5/6



Prototype for Lesson Construction



Quantity:

Concrete display of concept

Mathematical Structure

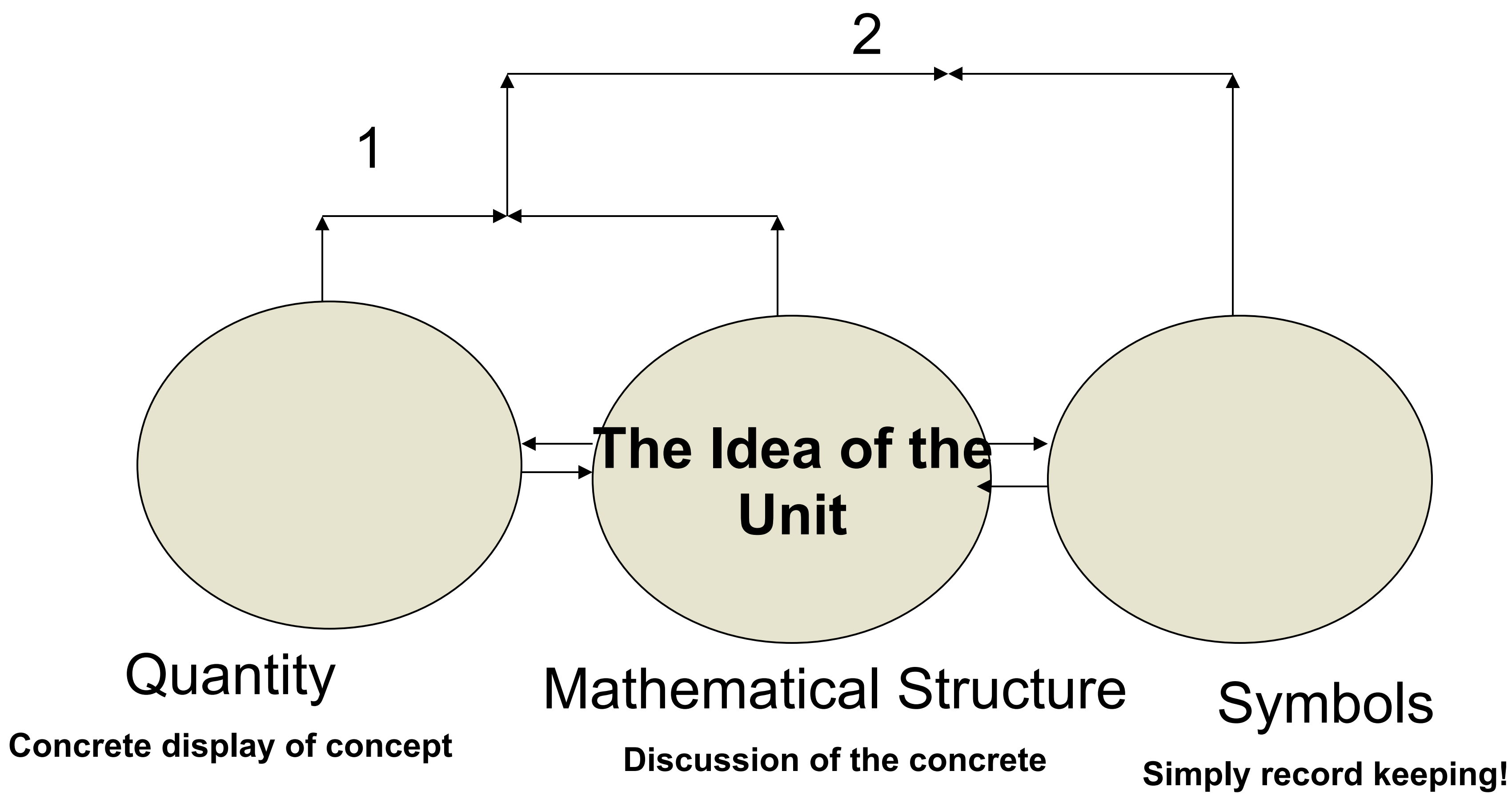
Discussion of the concrete

Symbols:

Simply record keeping!

V. Faulkner and DPI Task Force adapted from Griffin

Prototype for Lesson Construction



V. Faulkner and NCDPI Task Force adapted from Griffin

Number and Operation – Fractions[1] 4.NF

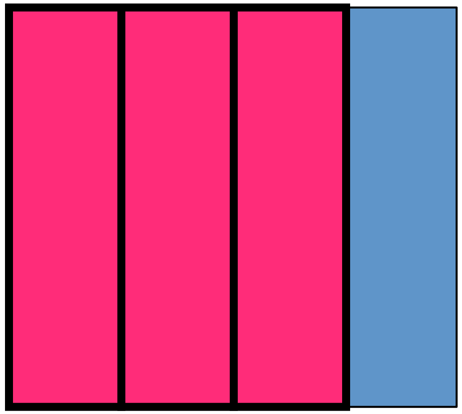
Common Core Cluster

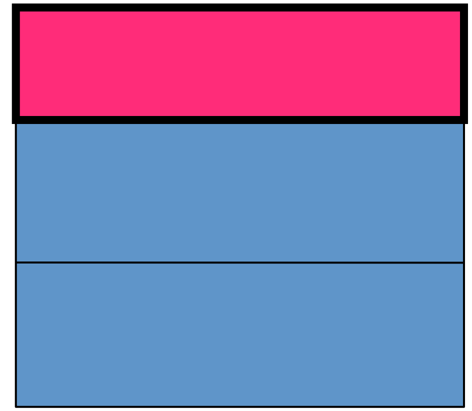
Extend understanding of fraction equivalence and ordering.
Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **partition(ed), fraction, unit fraction, equivalent, multiple, reason, denominator, numerator, comparison/compare, ‹, ›, =, benchmark fraction**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
4.NF.1 Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	This standard refers to visual fraction models. This includes area models, number lines or it could be a collection/set model. This standard extends the work in third grade by using additional denominators (5, 10, 12, and 100) This standard addresses equivalent fractions by examining the idea that equivalent fractions can be created by multiplying both the numerator and denominator by the same number or by dividing a shaded region into various parts. Example:  **1/2 = 2/4 = 6/12** Technology Connection: http://illuminations.nctm.org/activitydetail.aspx?id=80
4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are	This standard calls students to compare fractions by creating visual fraction models or finding common denominators or numerators. Students' experiences should focus on visual fraction models rather than algorithms. When tested, models may or may not be included. Students should learn to draw fraction models to help them compare. Students must also recognize that they must consider the size of the whole when comparing fractions (ie, ½ and 1/8 of two medium pizzas is very different from ½ of one medium and 1/8 of one large).

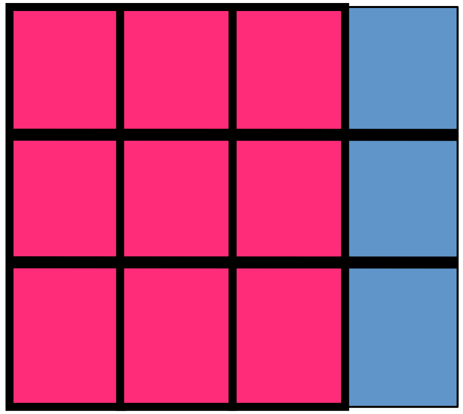
Lee Stiff’ s Unit Squares for Adding Fractions

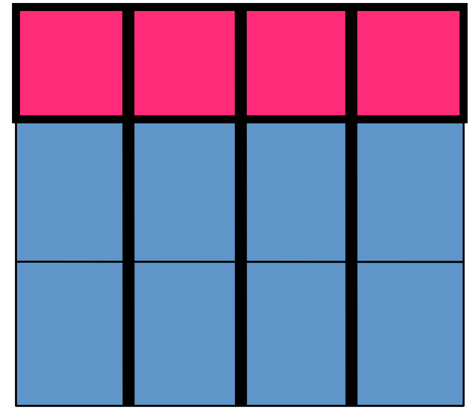




$\frac{3}{4}$ + 1/3

We’ ve got a problem in this form— need the same size pieces to add things…



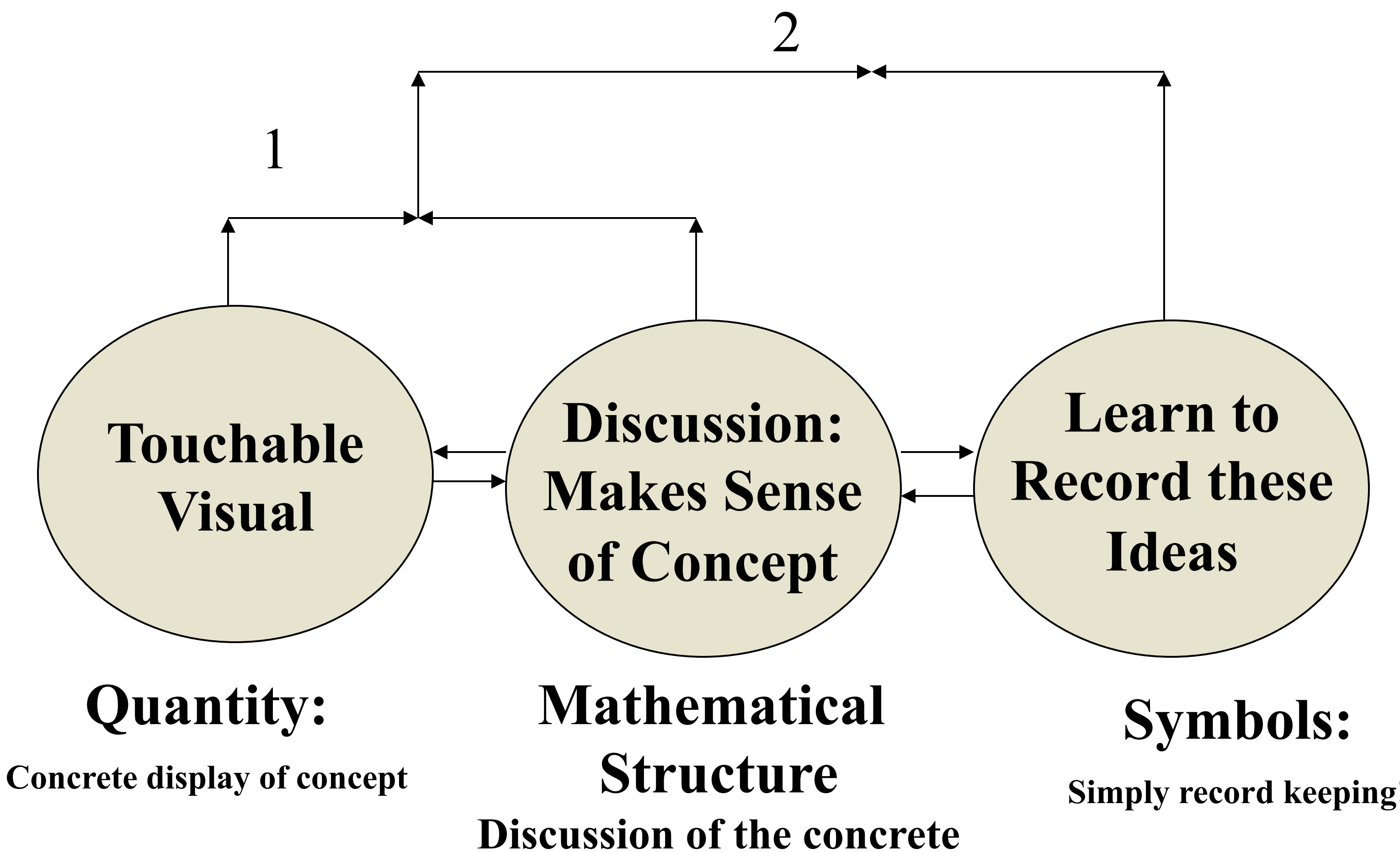


$\frac{3}{4} + 1/3$

$9/12 + 4/12$

Chop up the vertical by the horizontal
and the horizontal by the vertical:
Don’t change the value, just the piece size

Prototype for Lesson Construction



V. Faulkner and DPI Task Force adapted from Griffin