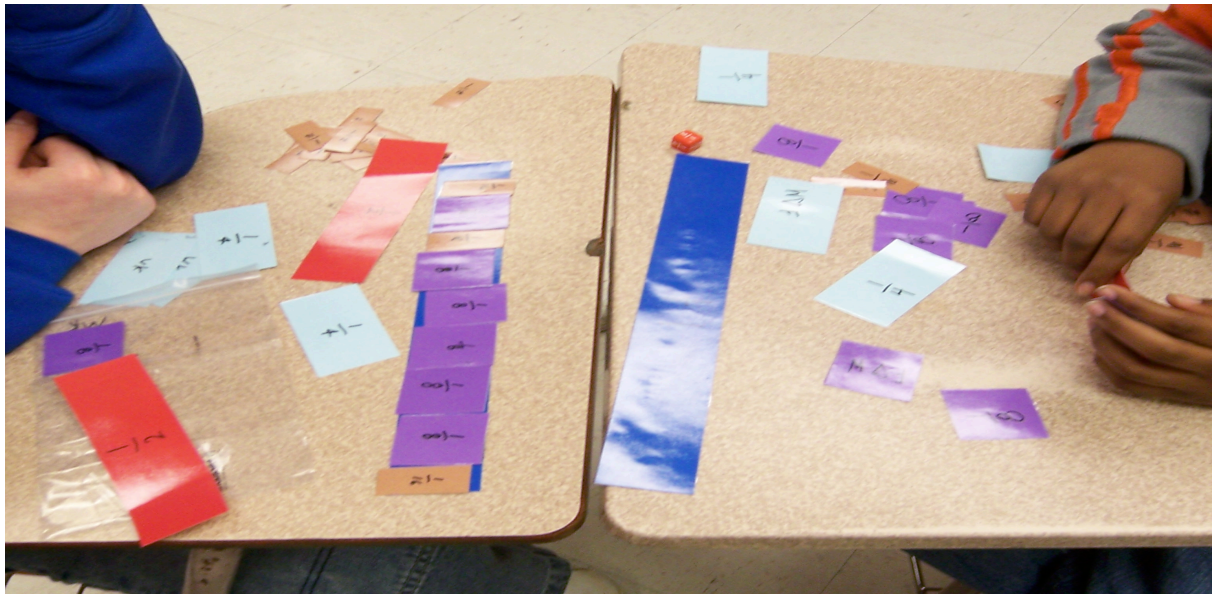
Marilyn Burn's Fraction Kit
MathSolutions.com



Cover-up

Uncover 1

Equivalent fractions

and compare fractions

Dozens more activities throughout the warm-ups.

Presented in an order that builds and connects over time.

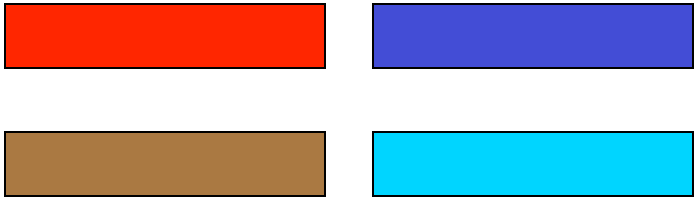
Marilyn Burn's Fraction Kit

Fraction Friday Week 31

Fractions and Quotients

Picture Form	Fraction Form	Division Form	Decimal Form
	_____	_____	_____
	_____	_____	_____
	_____	_____	_____
	_____	_____	_____
	_____	_____	_____

Fraction Friday



- Fraction Kit Week Four

- **Teacher Prep**
  - Read p.3-4 (paragraph 10)
  - Copy page 25 for class
- **Class time**
- Have students get kit and distribute as discussed. *If there are any problems, practice again.*
- Play Uncover Version 1

Middle School Resource and Inclusion Math

WCPSS Warm-ups

Valerie Faulkner

Kathy Sargent

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Valeriefaulknermathclub.com

- Calendar and Conference Presentations

High Achieving countries implement connections problems as connections problems

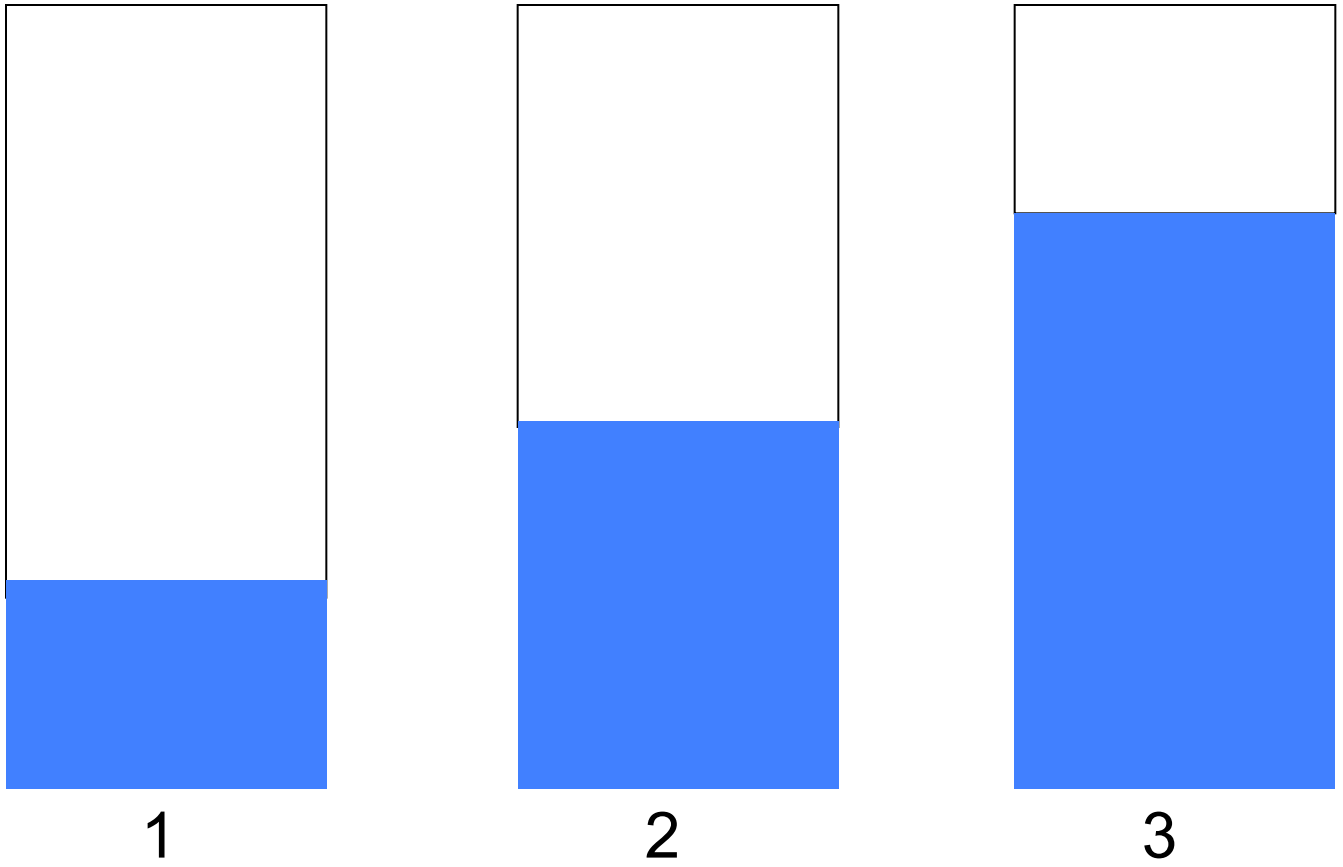
U.S. implements connections problems as a set of procedures

When math is presented

procedurally

what happens to the brain?...

The Accumulator Model: Our Analog Brain



Stanislas Dehaene 1999. The Number Sense


TWO WAY POWER
6.55
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Euro
MRC
M−
M+
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8
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RATE SET
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9:30

Gas Gauge picture From url:
http://www.marinepartssource.com/newdetails.asp?mfgno=57902P&pnumber=S57902P&mfg=TELEFLEX&desc=Amega%20Fuel%20Gauge

.78

Fuel Level

http://www.americanflyers.net/aviationlibrary/instrument_flying_handbook/chapter_4.htm

4.125 3.67 8.00

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36,482 1400 9

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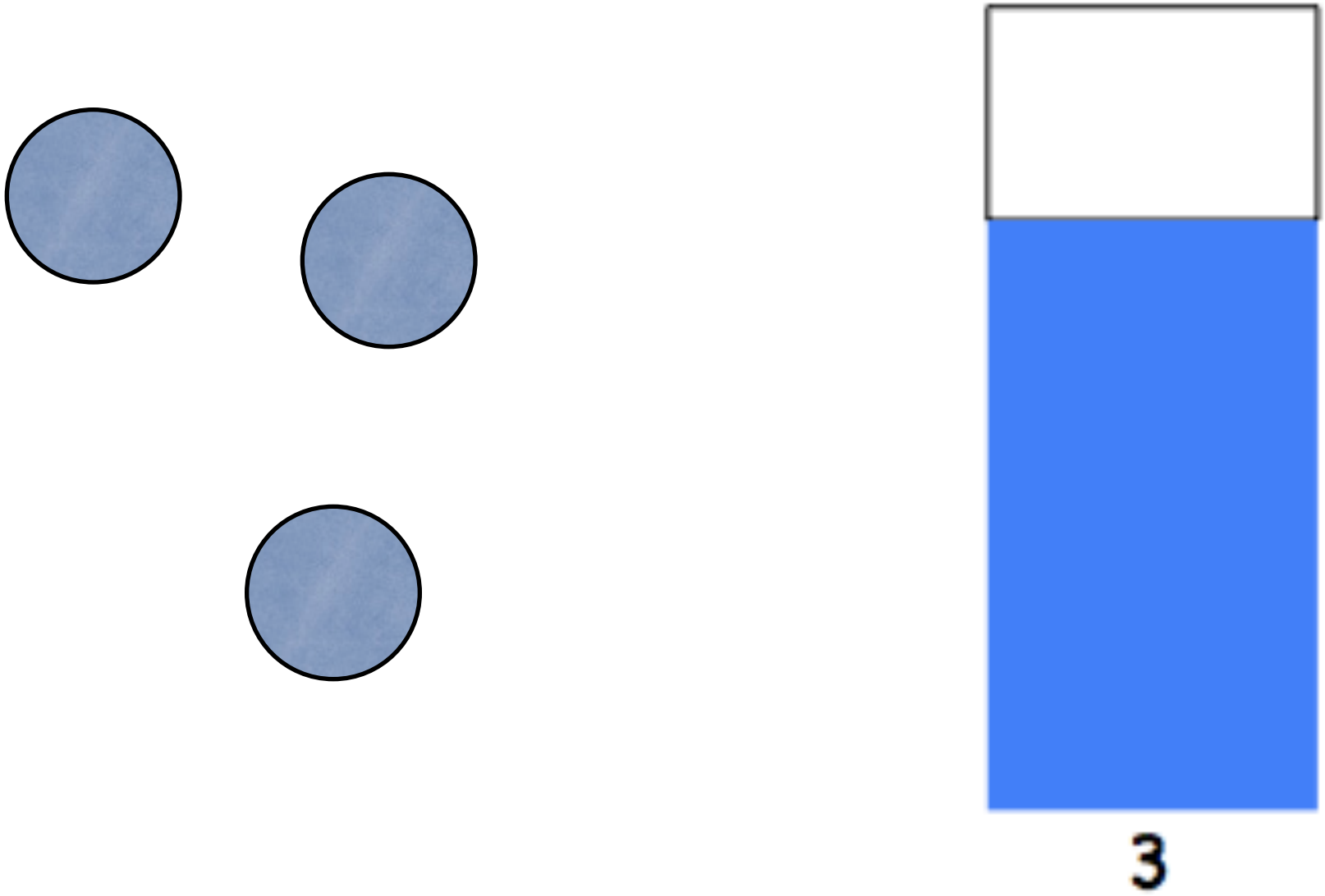
3

Cat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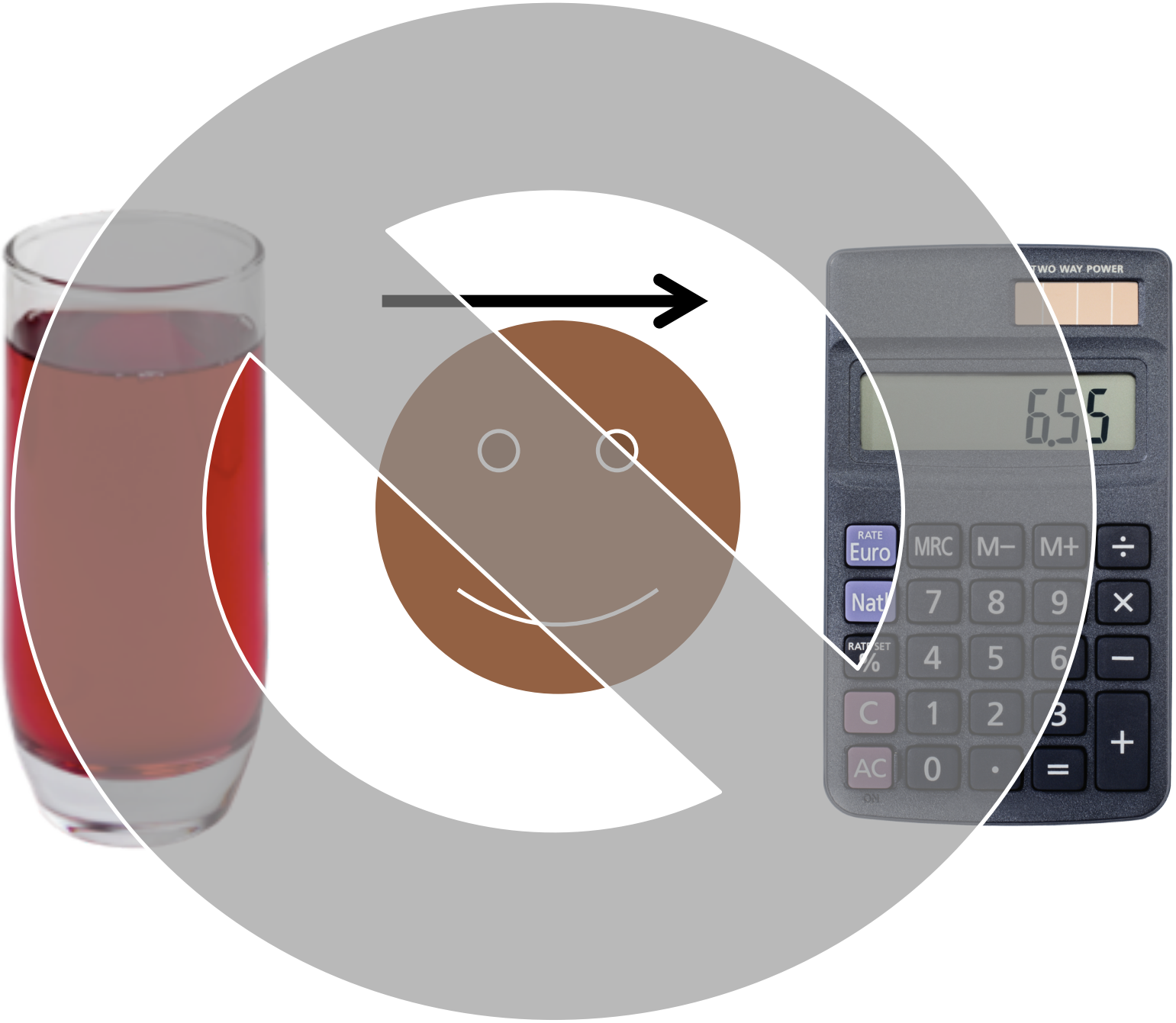
TWO WAY POWER
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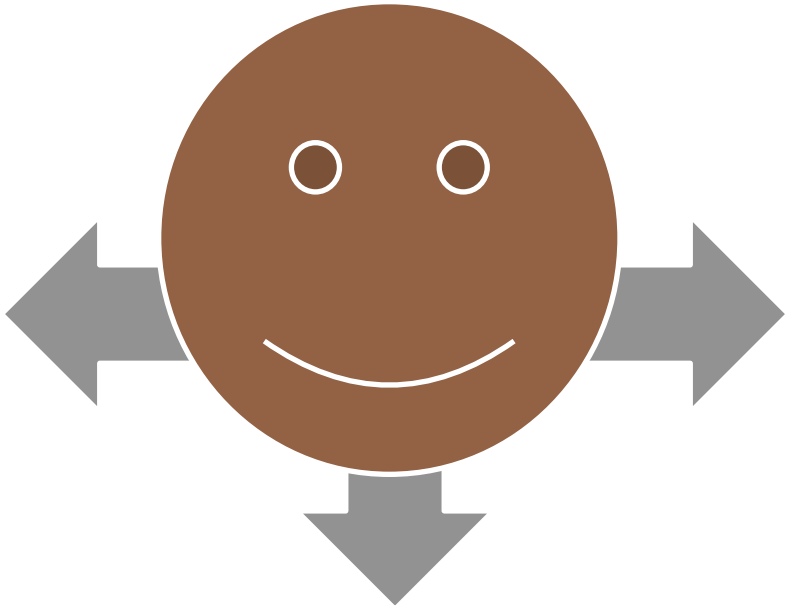


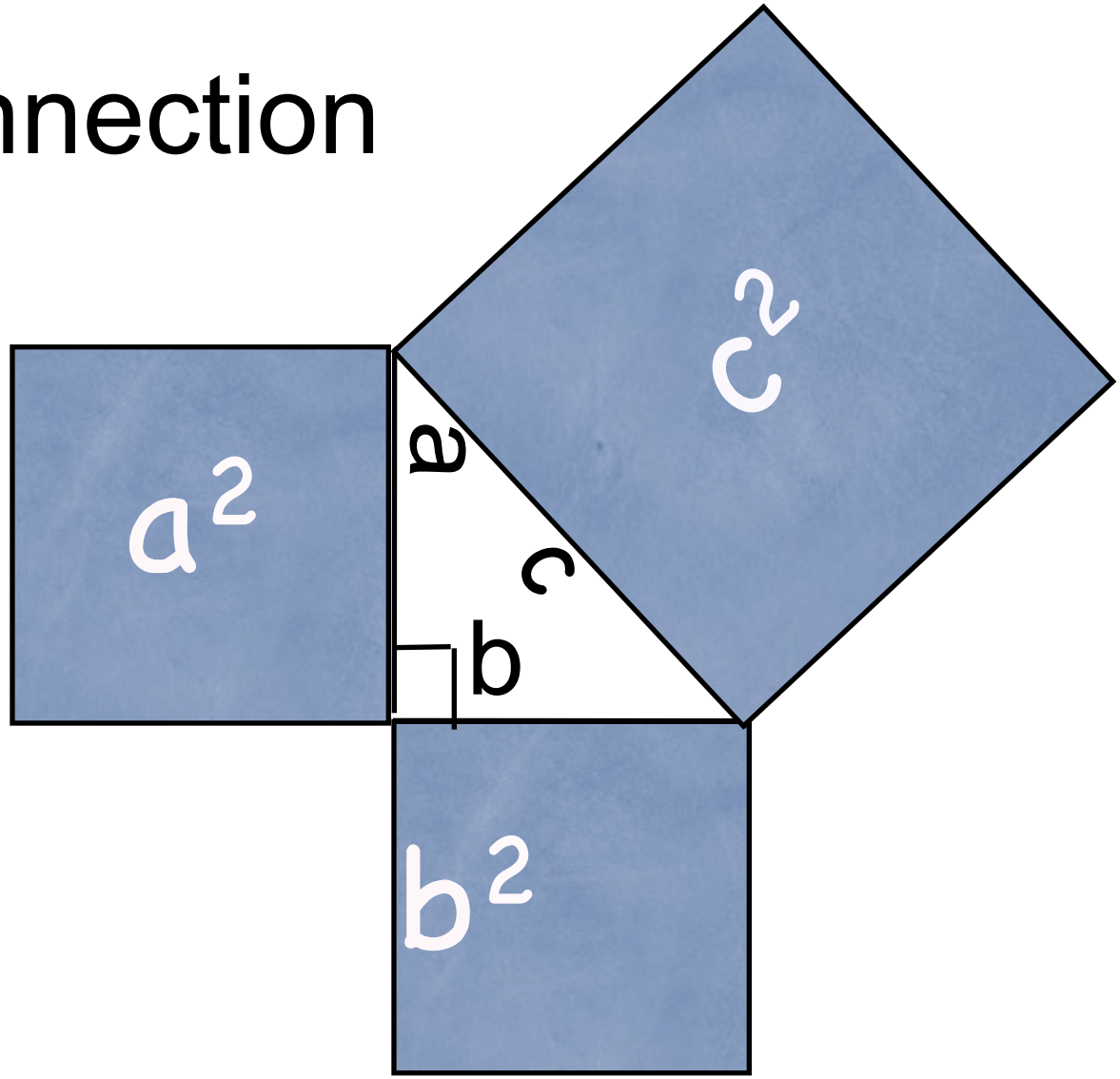
There were 10 glasses of water and I drank a little less than 3 ½ glasses.

There were 10 glasses of water and I drank a little less than 3 ½ glasses.

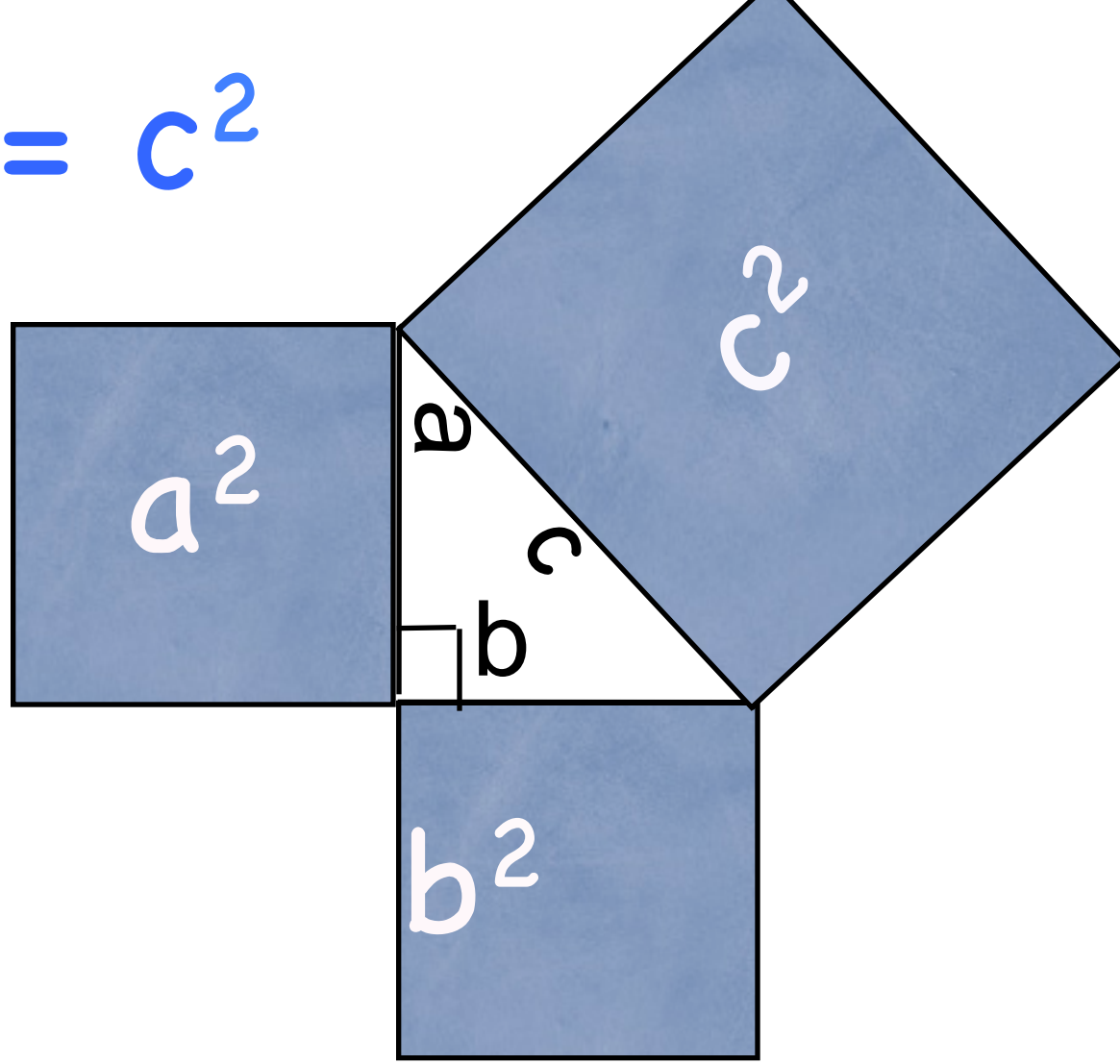
Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Pythagorean Theorem: Analog Connection



$$a^2 + b^2 = c^2$$



Cat

The role of the mathematics instructor:

Connect the analog to the digital…

The role of the mathematics instructor:

Connect the structural to the symbolic…

The role of the Common Core:

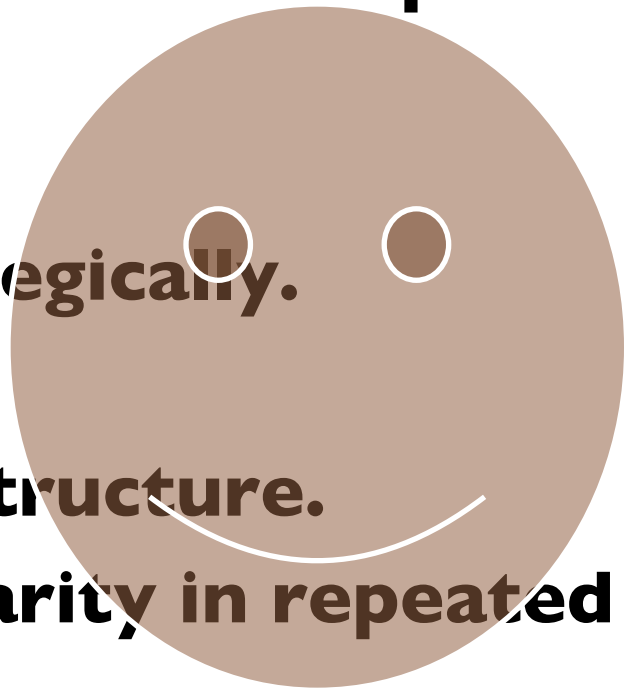
Support teachers in developing students who use mathematics to make sense of the world.

The role of the Common Core:

Support teachers in developing students who think critically, justify a claim, critique ideas, adjust their thinking, persevere, visualize and discuss.

Common Core Practice Standards

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**



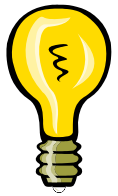
Talking is not only a means of learning…
talking is what we are teaching them to do.

Deborah Ball,
Dean of College of Education
University of Michigan
August 2012

The Role of the Warm-ups?

- Support Teachers in
  - Making connections across content
  - Teaching kids to TALK
- Teaching kids
  - to make connections
  - To think deliberately and mathematically

Exponents and Geometry



What is 4^2 ?

Why is it 4x4 when it looks like 4x2?

It means

'make a square out of your 4 unit side'

Exponents and Geometry

What is 4^2 ?

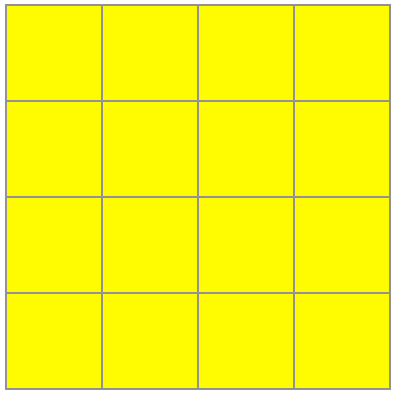
--4 units--

1

1

1

1



You'd get how many little 1 by 1 inch squares?

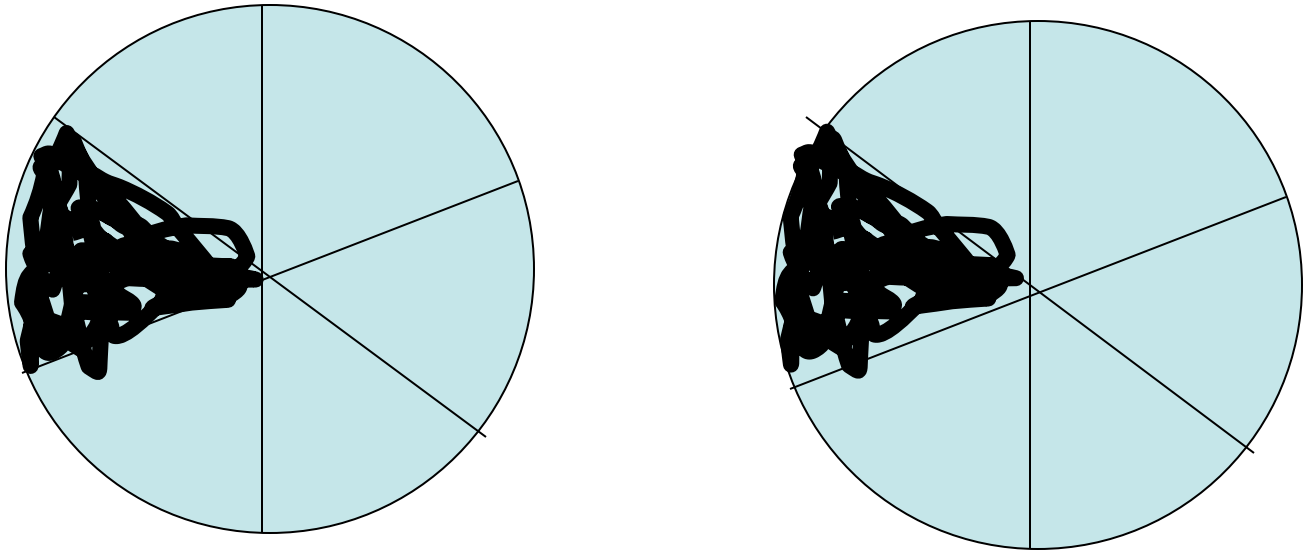
$4^2 = 16$

$$1 = 1^3$$

Are these the same?

Manipulatives and "Magical Hopes"

Deborah Ball—



1/6 + 1/6 = 2/6

Or is this 1/6 + 1/6 = 2/12? How do you explain this to a student?

Unit Size

3 ones and 2 ones

3 tens and 2 tens

3X and 2X

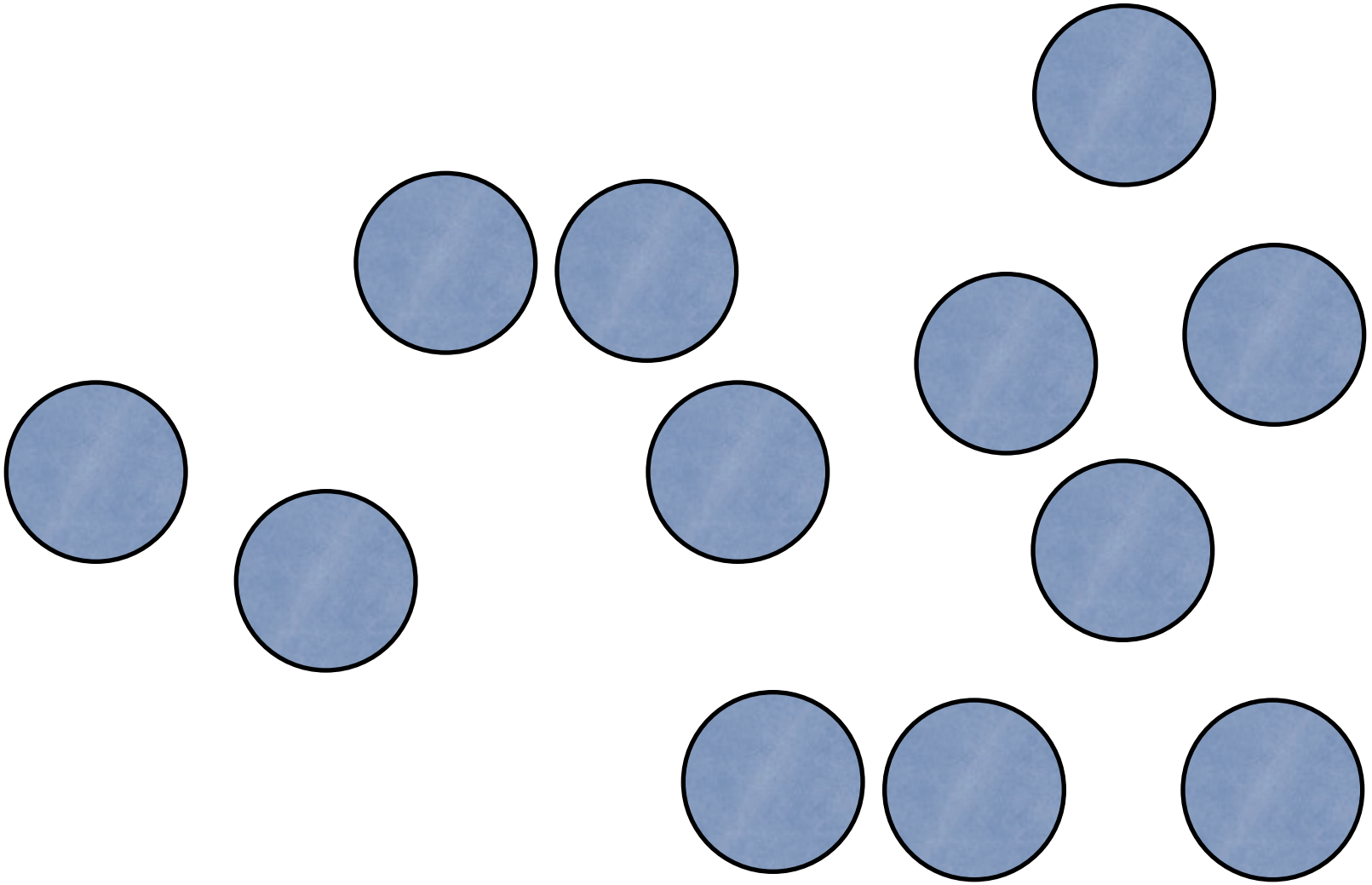
3 tens and 2 ones

3Y and 2Y

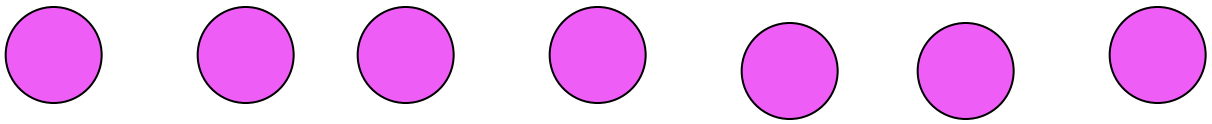
3/6 and 2/6

3/6 and 2/5

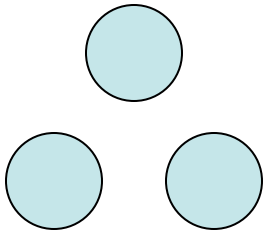
3X and 2Y

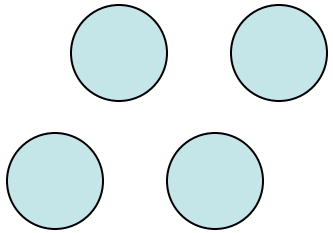


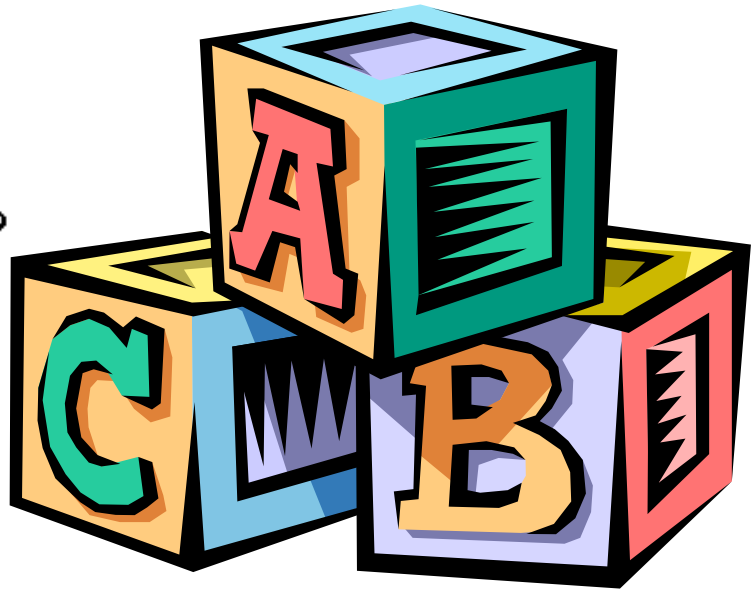
‘ones world’…



Numbers are composed of other numbers…







week-by-week Set 1 Resource Math Remedial Warm-ups & Assessment

Contact: Valerie Faulkner vfaulkner1@wcpss.net (919) 858-1599

User's Guide

- **This is to be used as a Tool for a limited portion of class-time only!**
  - Limit these activities to the first 10-25 minutes of class (depending on length of class– for instance: 10 minutes for a 45 minute class; 25 minutes for an 80 minute class)
  - This warm-up notebook is designed to guide the teacher through developing the students' concrete understanding of the number system and thus remediate difficulties and develop number sense.
- Students who are taking the EOG or Extend2 **must have access to the SCOS on or near grade level.**

Overview: Resource Math must include

- Remediation: Deliver support through daily warm-up that addresses elementary skills in a concrete and repetitive way.
- Daily Repetition of **concrete concepts**!

- SCoS: Deliver access to the standard course of study through daily lessons that follow structured plan (ex: Project achieve lesson plan outlines)
- Daily Access!

Rules and Expectations for Manipulatives

- Note that the first day of any unit that involves new manipulatives should be spent on reviewing where they are kept, how they are distributed, etc.
- Practice how this works with students.
- This may take an entire 20-30 minute session in the beginning.
- That's fine! Take the time up front to develop the expectations.
- Continue making this a primary emphasis in lessons through the first 9 weeks (at least)

Warm-ups: suggested time-line for Set 1

- Fall
  - Addition & Subtraction
    - Unit cubes and 10s rods
    - Money
    - 100s board
  - Base Ten
    - 100's board
    - 10s rods
    - Money
  - Equality and renaming numbers

- Spring
  - Multiplication
    - Units cubes and money
    - Arrays
    - 1x1 tiles
    - Partial products
    - Composite v prime
    - Squared numbers
  - Division

Fall and Spring:
Fraction Fridays
Fraction Kit (MathSolutions)

Teacher Note on Student Buy-in

Students know when they are working on remedial skills and often seem to feel bad about themselves for not getting it already. This sometimes seems to stir up resistance in the students to the task at hand. You may want to try and 'cut this off at the pass' by giving them a speech something like this, right up front:

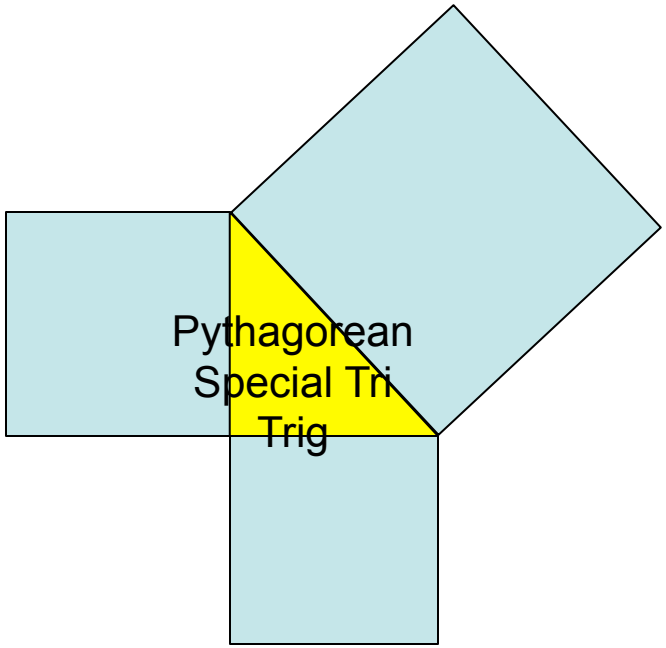
We've got a lot to learn this year and, as you know, we'll be working on 6th grade SCoS material for the second half (or more) of every class period. But we're also going to do something that is just as important—we're going to investigate how our number system works. Believe it or not, Mathematicians study the number system all the time and even have a name for it—'Number Theory.' So what we're doing, especially in the first semester, LOOKS like we're doing things you've done before, but what we're really doing is learning how the system works. People may not have explained the number system to you before, but now that you're in middle school, you really need to understand this so that you can go on to algebra and learn to really think like a mathematician. You need to trust me here, and I think you'll enjoy ***understanding***

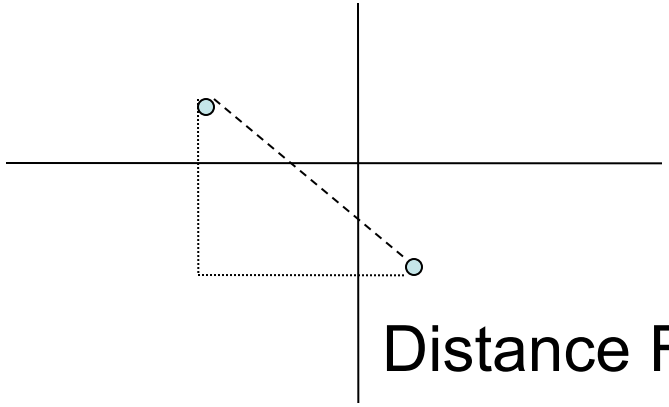
Exponents and CONNECTIONS

Square Roots! $\sqrt{16}$ = 4

The length of one side!

Geometry and Measurement





Distance Formula

Materials List

1st 9 weeks

- Base ten blocks
- Fake Money

Journal Activities

- Week 1—Why do you think we group things by ten?
- Week 2—Would you rather have a ten or ten ones in your wallet and why?
- Week 3—Explain what the number words eleven, twelve and thirteen really mean.
- Week 4—What is a double?
- Week 5—Explain the number 10: What does the 1 mean and what does the 0 mean and how do you know?
- Week 6— If you owed a friend five dollars and you had a ten dollar bill, what would you need to do?
- Week 7—If I have a ten dollar bill and someone else has 2 one dollar bills, how could I show that the ten is really worth more money?
- Week 8—Write a subtraction word problem and then show the solution for the problem.

Week 1: Addition

Concepts:

- **Develop idea of grouping by 10**
- **Develop idea of Equal Exchange**
- **Explain nominal system and number system**

Strategies:

- Support counting all but demonstrate and encourage counting up from bigger number

Materials:

Unit cubes (18 per student)

10s rods (1 per student)

Paper cups (optional)

Lessons:

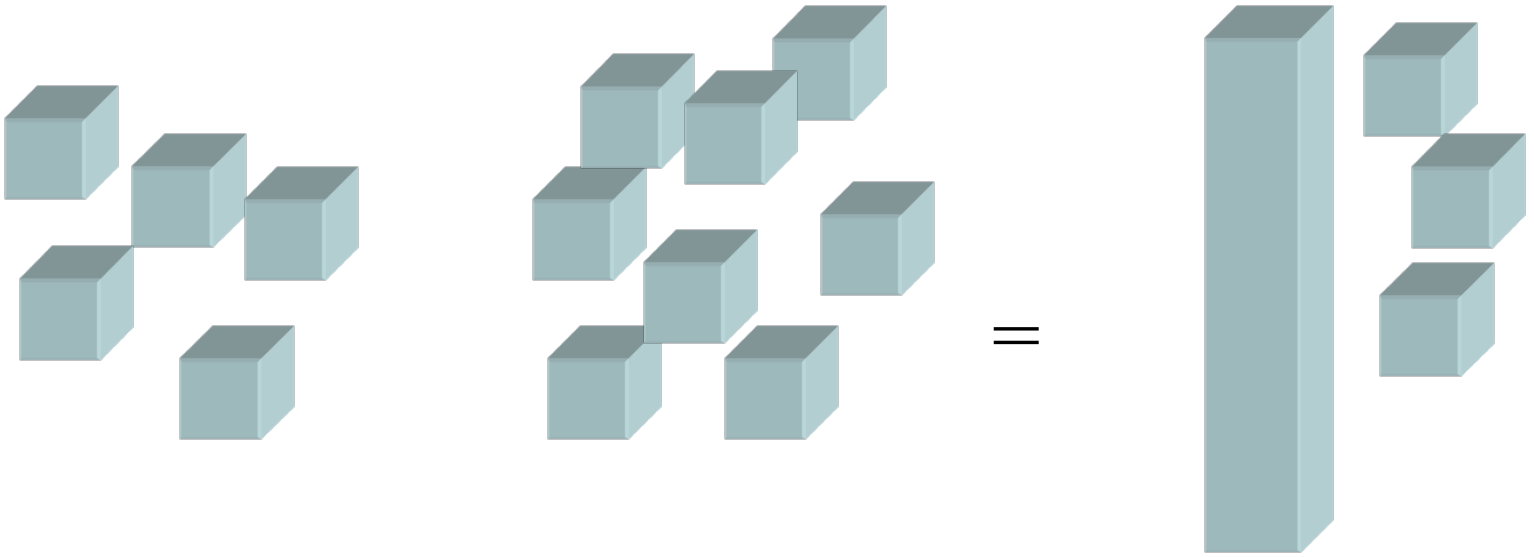
FIRST: discuss how you will distribute and use the math manipulatives. Practice this with students and go through stages with them as needed (model and rehearsal, etc)..

THEN Go through teaching stages for the mathematics: Model, Guided Practice, Working in Pairs, Independent Practice.

Present problems such as 5+8. Have students represent 5 and 8 in separate piles. Then have students group by counting up from 5 or 8. Put group of ten to the side or in a cup. Trade in group of ten for 10-rod and discuss result. Discuss the idea of 1 ten and 3 ones. On the board, record for the students what you have done: 5 + 8 = 13

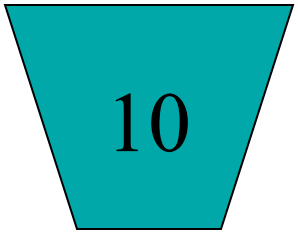
NOTE: Be sure to include problems where a ten is not made (3+4) and discuss why you do not need to trade in for a ten rod.

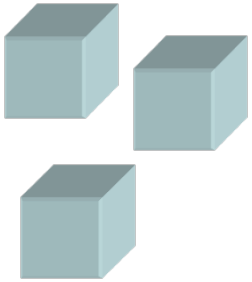
Week 1: Addition—What it looks like



13

or





5 + 8 = 1 ten and 3 ones =

$$\begin{array}{r} 5 \\ +\ 8 \\ \hline 13 \end{array}$$

Week 1: Addition--Notes

- Keep your examples “within 20” (sums no more than 18).
- Give one “non-example” for every 6 or 7 problems (problems that do not need regrouping like 4 +3).
- Examples:
- 2 + 9 9 + 2 6 + 8 7 + 9 5 + 2 9 + 6

- Notice if students are “counting all” or “counting up”
  - For those that are still counting all, Support them for their accurate and successful strategy. At the same time, see if you can get them to start with either pile and name that number and count up from there. This more sophisticated strategy can generally be developed with time.

Week 1: Addition—Example Dialogue

1) Emphasize how we group things by 10 in our number system and how numbers & names work.

- 5 + 8 "Now group ten together and trade that in for a 10-rod…Now what do you have? A ten and 3 ones left over. Good. That's how our number system works. We always group with tens. We write that number 1 3 to mean one ten and 3 ones. In chinese they call that number 1 ten and 3 ones, but we call it thirteen. Can you kind of see how our word has ten and three in it? It's more confusing, but that's where the word comes from"
- 2 + 9 " Now group this problem into a ten and left overs and trade your group of ten in for a 10-rod. What do you get? 1 ten and one left over one. Again in the Chinese language this would be 'one ten and one one' but we have the annoying name of eleven for this number. We write it just like the Chinese people would say it though-- 1 1.
- 4 + 8 Let students represent this problem and then group the numbers into a ten and 2 ones. "What do you get when you trade your group of ten in for a 10-rod? A ten and 2 ones left over. What do you think the Chinese language says for this number? 1 ten and 2 ones. Good. What do we call it? Twelve. Good. Eleven and twelve are the most difficult names for our numbers. If you can remember them and think about what they **really mean** (1 ten and 1 one; 1 ten and 2 ones) , you're in great shape".
- Try this one. 8 + 7. " what did you get? A ten and 5 ones. Good. Now what is that name in our language? Fifteen. Can you kind of see the ten and five ones in that name? It's backwards and annoying, but it's there. Good."

Week 1: Addition—Example Dialogue

2) Emphasize the idea of an Equal Exchange and Composing tens

- 8+8 " What did you get for this problem? A ten and 6 ones. Good. That's 16. Let's look at what we did when we traded in those ten ones for a 10-rod. Why did we do that? (to have less extra pieces, to simplify things, because our number system works in tens). Why is it okay for us to do that—In other words, did anything get cheated or why was it fair to do that? (both are equal). This is a very important part of mathematics. We need to be able to see when things are equal and we need to trade equal things a lot of times to get where we want to go. We build or 'compose' groups of ten so that we can trade them in for a ten.
- 9+9 "What did you get for this problem? A ten and 8 ones. Good. Why could you trade in the ten ones for a ten cube? Are you sure you can do this? Why? Are you sure it's okay?" Get them to see that this is not magic but just a logical trade based on things being equal.
- 5+4 "What did you get for this one? Could you trade in for a 10-rod? Why not? But that's a lot of ones that are sort of annoying, is there anything you can do with them? No, we're stuck with nine. We cannot trade in until we get a group of ten. So 5+ 4 is just 9. No tens just ones. So, notice that you cannot always trade in for a 10-rod. When can you trade? (only when you can compose a group of ten because that will <u>equal</u> a 10-rod)"
- 8+3 5+5 7+2 9+6 Etc.

Week 3: Addition

Concepts:

- **Continue to develop Week 1-2 goals of Base ten, equal exchange, recording your work etc.**
- **Develop concept of commutative nature of addition**

Strategies:

- Support counting all; encourage counting up

Materials:

Fake money per student:

18 1 dollar bills

1 10 dollar bill

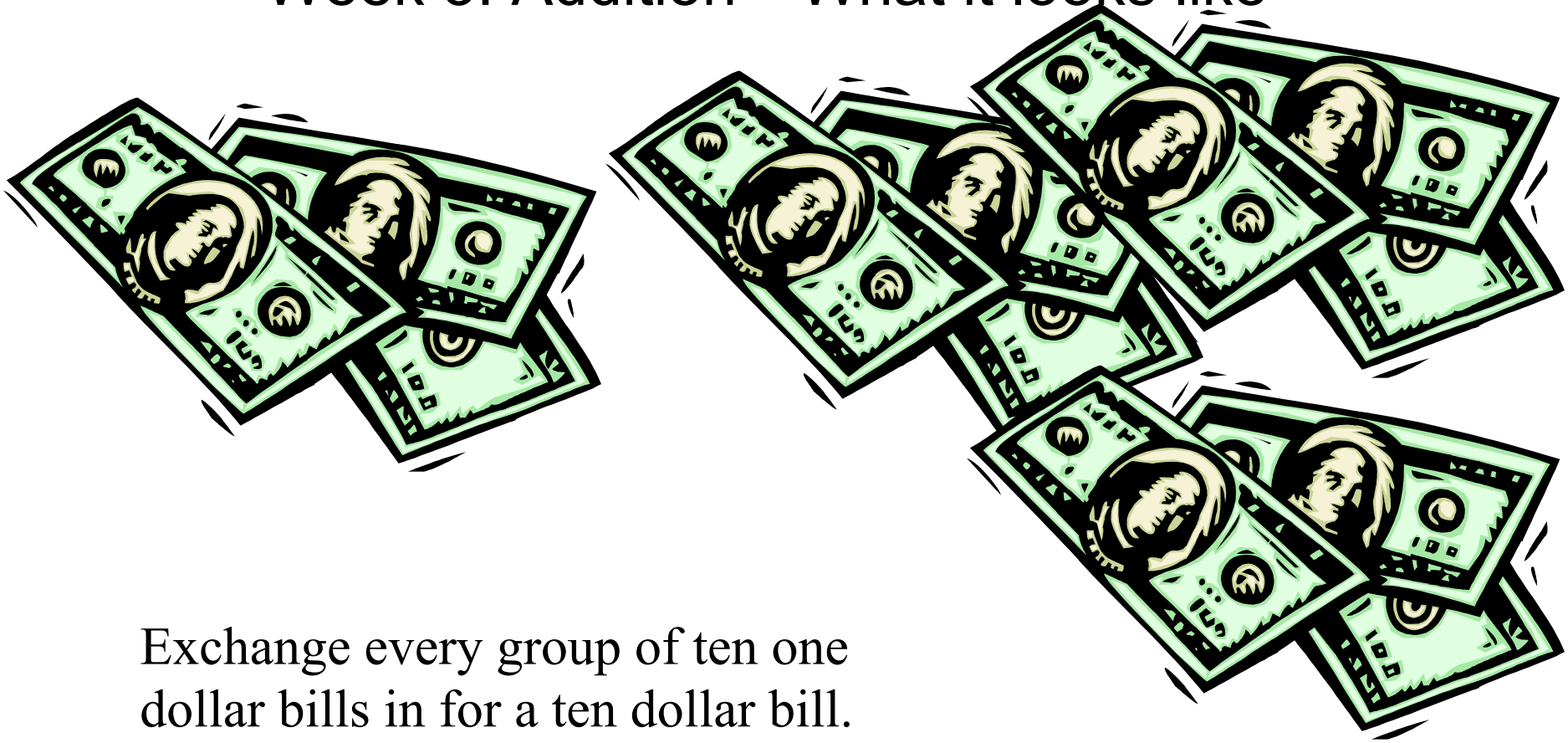
Lessons:

Go through teaching stages for the mathematics: Model, Guided Practice, Working in Pairs, Independent Practice.

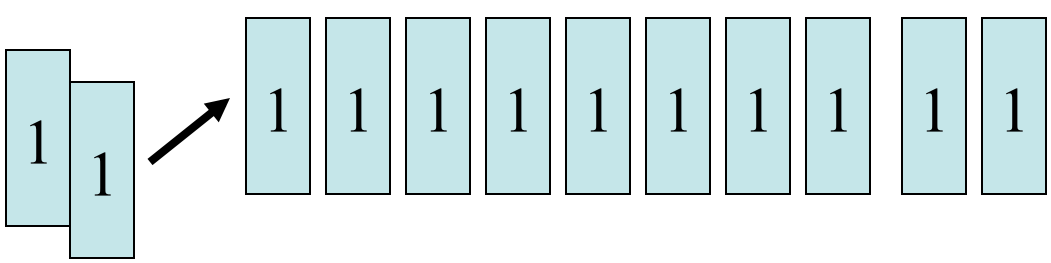
Continue to present problems such as 5+8. Use Money instead of the unit cubes and **point out that they are doing exactly the same thing.** Notice that you can really emphasize language that they can 'hold on to' because they are familiar with money. Develop concept of commutative nature of addition through discussions during week.

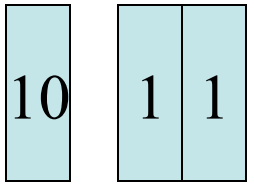
NOTE: Be sure to include problems where a ten is not made (3+4) and discuss why you do not need to trade in for a ten rod.

Week 3: Addition—What it looks like



Exchange every group of ten one dollar bills in for a ten dollar bill.





$3 + 9 = 1\ 2$

Week 3: Addition--Notes

- Use math language and more familiar money language to make connections
- Make sure students pile up their ten ones and then make a physical trade for a ten dollar bill.
- Emphasize making an equal exchange with the ‘bank’
- You can have groups of 2s wherein each student plays ‘the bank’ and becomes the accountant for the other student as they trade in their tens and total their money into sums.
- Include some discussion of how 7 + 3 ends up with the same total 3 + 7 by putting some problems like that in there side by side and then discussing it with the class (or pairs of students as you circulate around the room).

Week 3: Addition—Example Dialogue

5) <u>Develop and deepen understanding of Equal Exchange through use of money</u>

- 8 + 5 " Okay let's say you have 8 dollars and you earn 5 more, how much money do you have (pile of 8 and pile of 5 and then count all or count up)? 13 dollars. Do we have enough ones so that we could get a ten? How would we do that? We could go to the bank and trade in our ten ones for a ten. Why will the bank give us a ten? Are we getting 'ripped off'—I mean we're giving them 10 dollars and they are only giving us 1 ten dollar in return. Why is that okay? Right—it's EQUAL. Remember that's really important in math.
- Now the way this will work is that whenever you count up your money from a problem you then need to trade in for as few bills as possible using 1s and 10s. So, if you have 8 + 4 you would have how many ones? 12. Good, but is that the least bills you can have? No, you will need to trade in for a ten and you'll end up with 1 ten and 2 ones: 1 2. This is the 'standard form' of the number.

Week 3: Addition—Example Dialogue

7) <u>Continue to have students write down all problems. They will be the ‘Accountants’ for their money (either individually or in pairs like below):</u>

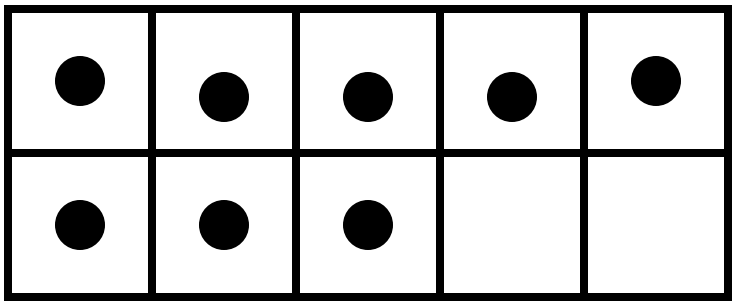
- “Okay, now that you see how to do it with money, and you see what we’re doing, I want you to resume writing down what you do. For each problem one of you will be the bank and the accountant while the other person works out the problem with the money. What is an ‘accountant’ by the way? Notice the word ‘count’ in there? An accountant is someone who keeps track of and ‘counts’ money for someone else. That’s what you’ll be doing. You’ll make sure that they give you a fair deal when they trade in for a ten and then you’ll keep track of the problem for them in writing by recording what they do. Change roles after each problem.”

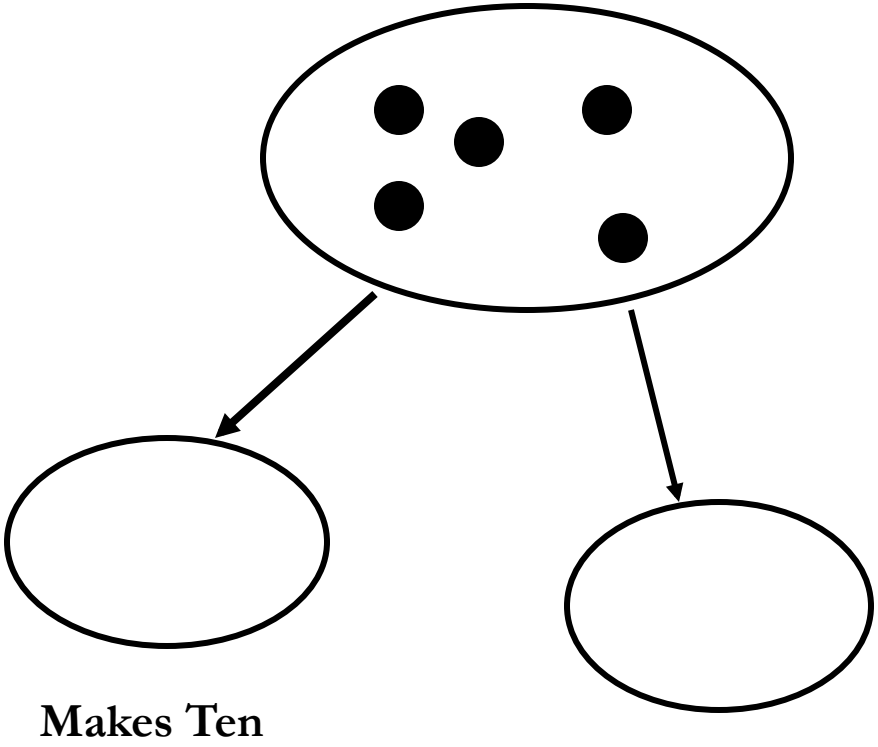
8) Encourage students to see that 7 + 3 yields the same sum as 3 + 7

- In those last two problems what did you notice? Good, for both of those problems we added the same numbers together and got the same sum. Does that make sense? Why does that work? Think about if you had 7 dollars and then found 3 more. Now think about if you had 3 and found 7. Those are two different problems and two different stories, BUT your total amount of money ends up being the same doesn't it? Because that works every single time in addition we have a name for it. It's called a Property when it works EVERY time, and that's the commutative property of addition. Notice the root word "commute" in there. What does commute mean? This works for addition but we'll see later it does not work for subtraction.

Making 10: Facts within 20

8 + 5





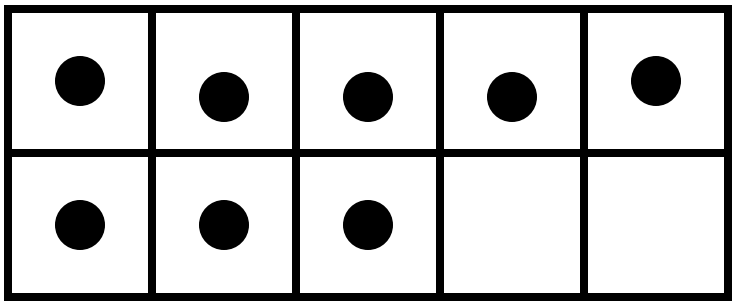
8 + (2 + 3)

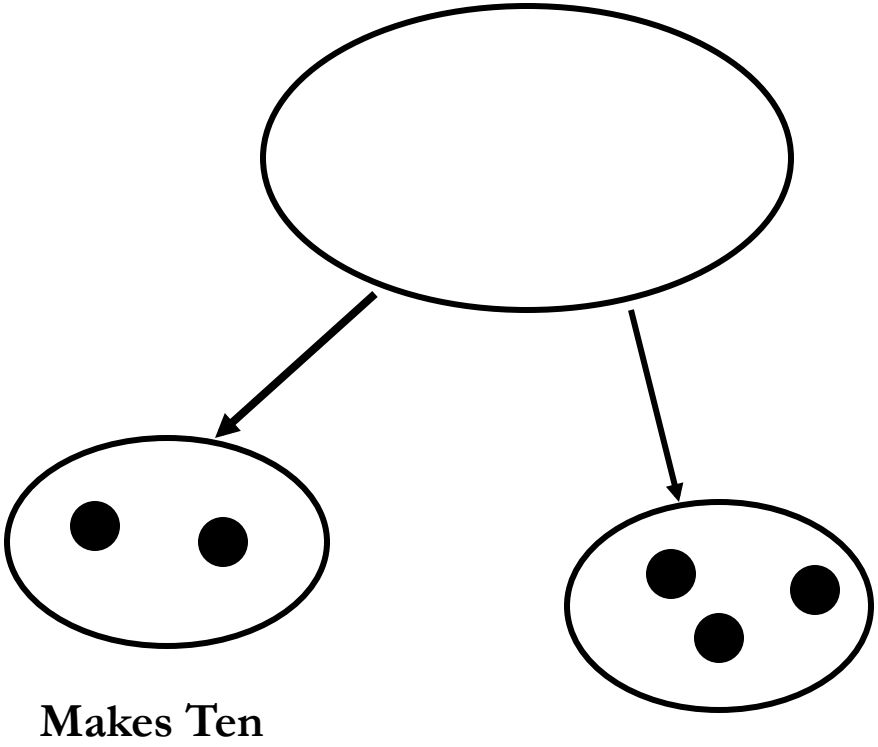
(8 + 2) + 3

1 ten and 3 ones (13)

Making 10: Facts within 20

8 + 5





8 + (2 + 3)

(8 + 2) + 3

1 ten and 3 ones (13)

Week 4: Subtraction

Concepts:

- **Continue with base ten concepts**
- **Model how the way we write this is just recording what we are doing with the cubes and rods**

Strategies:

Emphasize the physical strategies of exchanging and counting.

Develop the awareness of knowing when you need to make an exchange.

Materials:

Unit cubes (18 per student)

10s rods (1 per student)

Lessons:

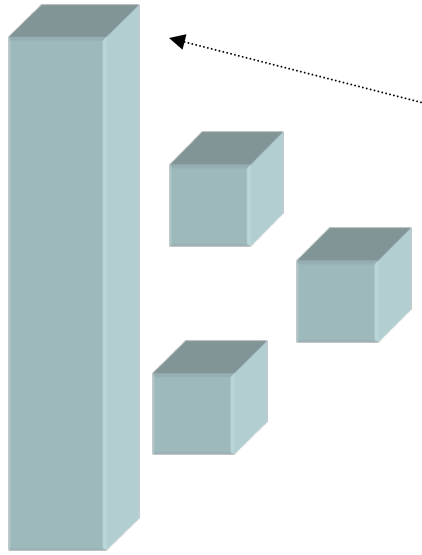
Go through teaching stages for the mathematics: Model, Guided Practice, Working in Pairs, Independent Practice.

Develop Subtraction with regrouping by relating it to addition. When we added we grouped ones together to made a ten. When we subtract we need to break down tens to get ones.

Continue to emphasize the importance of base ten (we group things at the rate of 10 in our number system) **and model the way we write this.**

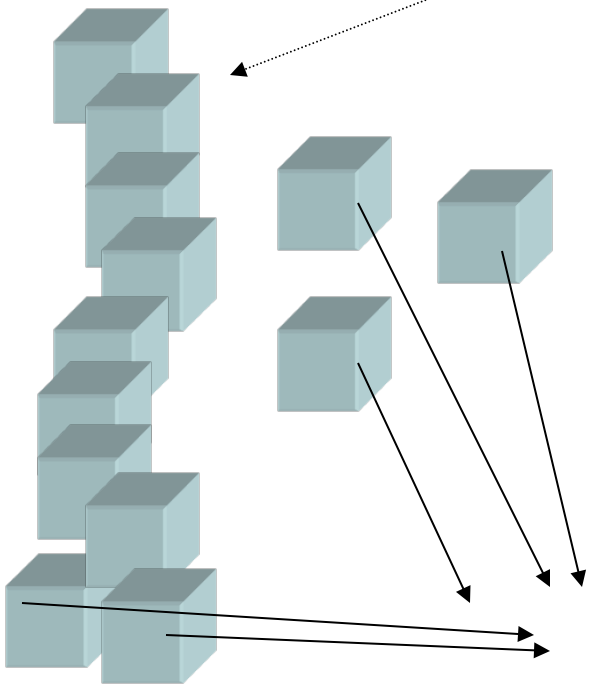
NOTE: Be sure to include problems where a ten does not need to be broken down into ones (14 - 3) and discuss why you do not need to trade in for ones. This develops metacgonitive awareness.

Week 4: Subtraction—What it looks like



\- 5

(13 cubes – 5 cubes)
Trade in (‘decompose’)
ten rod for ten ones



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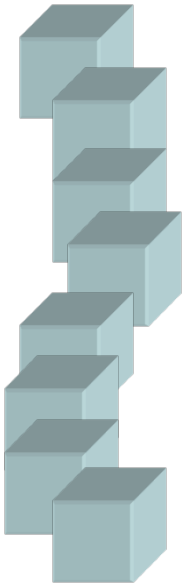


Week 4: Subtraction--Notes

- Keep your examples "within 20."
- Give one "non-example" for every 6 or 7 problems (problems that do not need regrouping like 15 – 3).
- Emphasize making an equal exchange and how it is the opposite of what we did in addition (instead of building or 'composing' tens we are now 'decomposing' tens when we need ones).
- Make a big deal out of the exchange—have students physically trade in the ten rod for ten ones, count out to make sure it's a fair/equal trade and discuss the fact that you still have the same value, just in a different form that will work better for you.
- **Keep all students engaged—for those who "know the answer" remind them that you are working on understanding the system, not on just 'getting the answer'**

Week 4:Subtraction—Example Dialogue

1) Make connection between composing tens and decomposing tens

- 16 – 7 First I make my 16 out of a ten and 6 ones—we always start with the number in this standard form (tens and ones) because that's how we write the number (tens and ones). Now how would we take 7 ones away from a ten and six ones? Let's see: 1,2,3,4,5,6… We don't have enough ones with the number in this form. But what can we do with this ten rod? There are ones 'in there' aren't there? Remember, in addition we put ones together to get a ten: we can do the opposite as well. So what can we do with this ten rod? Right, we can trade it in for ten ones. So let's do that—here's a ten rod, now let me take ten ones from the 'bank' and let's look at my number now. Now the number is in a different form. 1,2,3,4,…16. Okay good, we still have 16. Now I can subtract 7 and see what's left.

2) Demonstrate non-examples so that students develop the habit of thinking about the problem as their initial step for every problem

- 14-2 First I make my 14 out of a ten and 4 ones. Now, let's see. How can I take 2 away from a ten and 4 ones. Let's see: 1,2. Okay good, I can do that without needing to go to the bank. I have a ten and 2 ones left over so my answer is 12.

Week 4: Subtraction—Example Dialogue

3) Develop students' ability 1) to notice if they need to make an exchange and 2) to solve.

- Remember—set it up, check for exchange, solve. (or you and your class can come up with your own names for this Metacognitive system to remind students how to proceed. Give it three steps and have the middle step be checking to see if an exchange is needed).

4) Model in writing what we are doing with the cubes and rods

- Show students how what they are doing physically connects to the standard algorithm for subtraction:

Original problem →

$$\begin{array}{r} 1\ 3 \\ -\ 5 \\ \hline \end{array}$$

$$\begin{array}{r} \cancel{1}\ 3 \\ -\ 5 \\ \hline \end{array}$$

← We decompose a ten to get ten ones

$$\begin{array}{r} 0\ 13 \\ \cancel{1}\ \cancel{3} \\ -5 \\ \hline 8 \end{array}$$

This is modeled in our writing:
Notice how the 13 ones are in the ones column;
This represents a very literal recording of what we do with our hands. **Model it this way for Students!**

Subtraction within 20 Language Tips

- Standard Form and Ones Form
- Equal refers to value
- A ten rod is not the same as ten ones
- Chinese language: “1 ten 2 ones”
- Mathematicians evaluate the form—
  - Is this the form I want my value in?

Practice!:
Base 10 and Equal Exchange

16-4	14-7
12-8	17-3
13-5	19-9
18-9	15-6

Week 6: Subtraction

Concepts:

- **Connect base ten issues to money**
- **Continue development of base ten and equal exchange**

Strategies:

Deepen students understanding of number system and develop their flexibility.

Materials:

Fake money per student:

18 1 dollar bills

1 10 dollar bill

Lessons:

Go through teaching stages for the mathematics: Model, Guided Practice, Working in Pairs, Independent Practice.

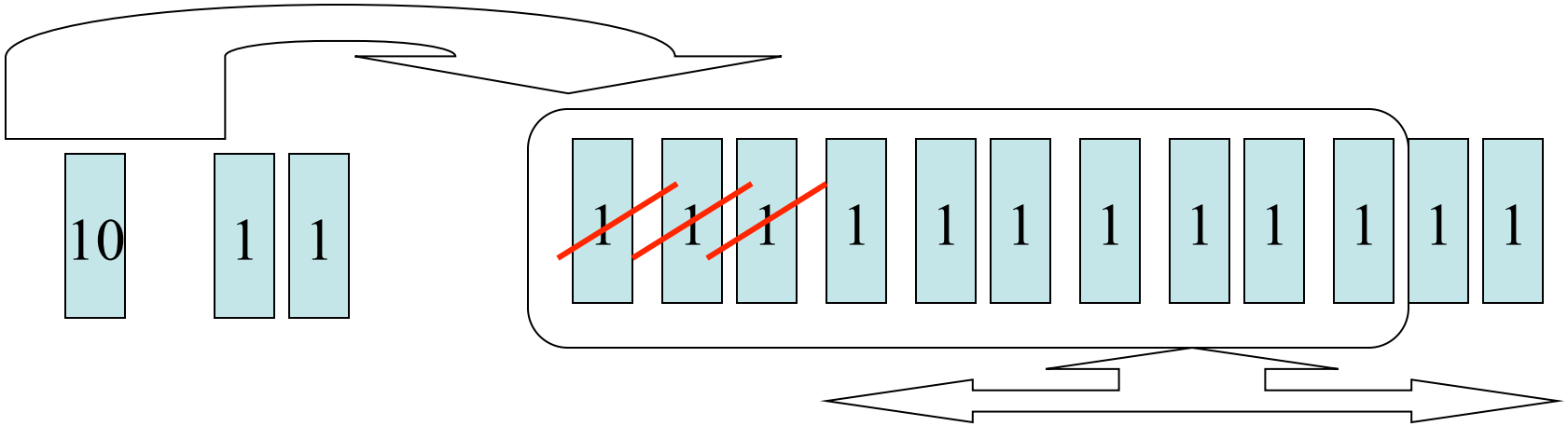
Have students make the connection between base ten and money

Continue to emphasize understanding the system of base ten, not just getting the right answer.

NOTE: Be sure to include problems where a ten does not need to be broken down into ones (14 - 3) and discuss why you do not need to trade in for ones.

Week 6: Subtraction—What it looks like

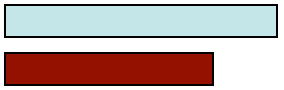
Exhange your ten dollar bill for ten ones



12 - 3 = 9

Week 6: Subtraction--Notes

- Emphasize the process of trading in the ten for ten ones and how this is a practical thing you do with money on a regular basis.
- Make it real with stories (you owe me money, how much more money does Tyrone have than Toneka, etc.)
- Make sure you understand subtraction deeply (not just the procedure)
- Note the 3 types of Subtraction problems--Discuss and include all types in your dialogue with students:
  - "Take away"
  - Comparison (difference between the two)
  - Missing amount (how many more do you need?)



Week 6: Subtraction—Example Dialogue

6) <u>Create stories to engage the students and emphasize how subtraction is just like what we do with money</u>

15- 7 Alright, let's say you have 15 dollars that you just earned mowing a lawn and you want to go to a movie tonight and it will cost $7. How much money will you have left over? Okay, first let's represent the 15 dollars in standard form (one ten and five ones). Good. Now, Let's see what happens when you spend the 7 dollars. Do you have enough ones to just give up your ones and keep your ten? No, you don't, correct. So when we work the problem, we need to break up the ten into ten ones and then we can give out the 7 ones. Let's do that both with the bills and then on paper. What do we have left after we go to the movies? Good, 8 dollars.

Connection:
Decomposing the Higher Unit Value

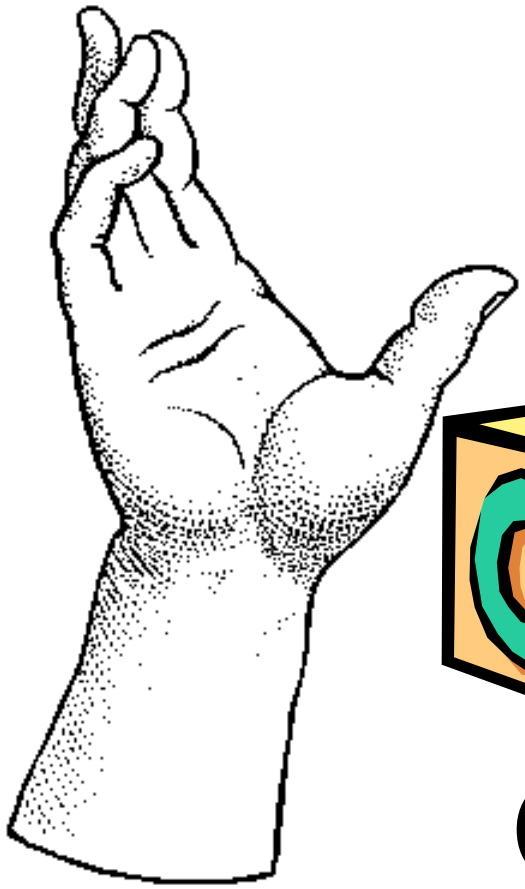
15 - 8

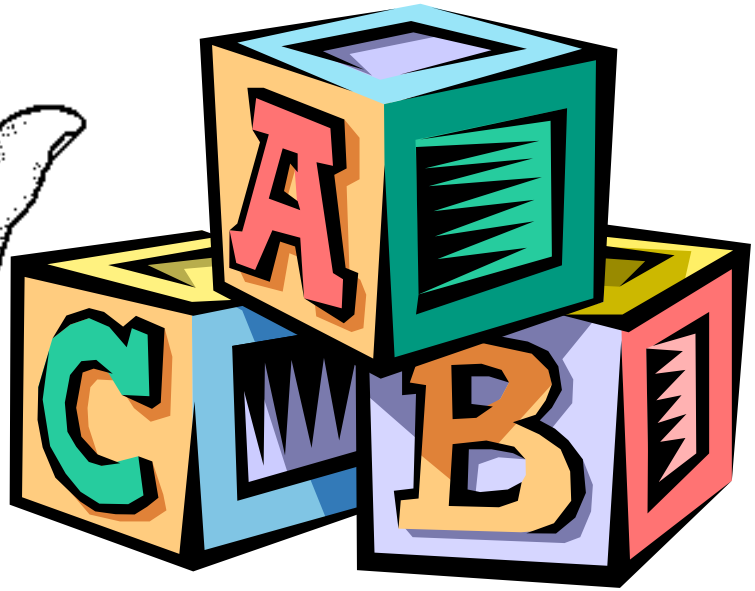
How do you teach facts within 20?

$1\ 3/8 - 5/8$

How do you teach problems such as the above?

Lunch





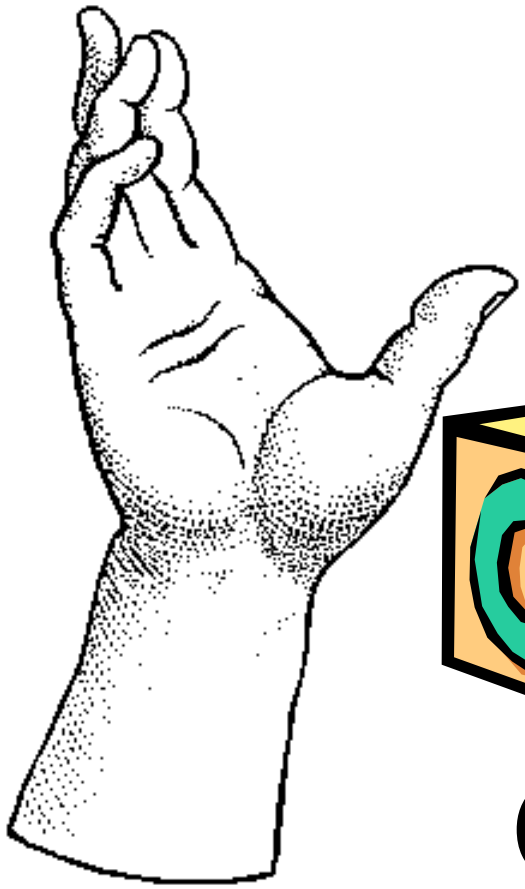
Quarter 2

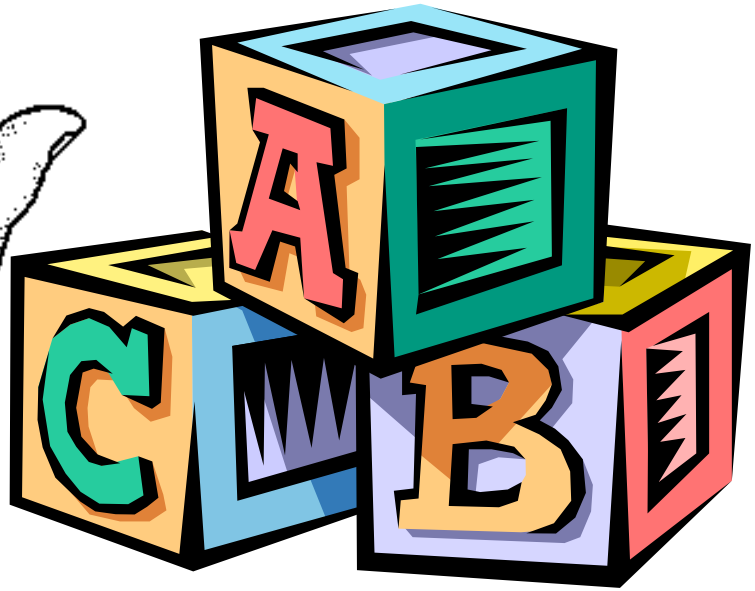
Set 1 Resource Math

Remedial Warm-ups & Assessment

Remove Quarter 2!

- Experience tells us…





Quarter 3

Set 1 Resource Math

Remedial Warm-ups & Assessment

Stand up and be Counted!

7s, 8s and 9s

Quarter 3: Multiplication

There is a lot to cover and connect in this nine weeks. See what your kids can handle. Plan to get through everything, and then use the last two weeks to adjust for your class—review the basics if needed, or work to extend the ideas.

There are so many important concepts here:

- the distributive property
- base-ten
- coefficients
- exponents
- the connection between exponents and geometry
- multiplying large numbers
- seeing the equivalent forms of a number.

Enjoy!

Implementation Notes:

As you will see as you teach this nine weeks, multiplication is really a key that unlocks so many connections in mathematics. I think you will see that there is so much more to learn than multiplication tables. The work here should be to tap into the math. These lessons are not designed to work on rote memory of multiplication facts. They are designed for students to understand what multiplication is and how it works in our number system. Memorization of facts and strategies to access facts can come later, as students feel empowered to want to know those facts. For lessons with larger numbers, or for those who really struggle with even the simplest multiplication, allow students to have a multiplication table or calculator so they can focus their attention on the concepts involved with multiplication.

Vocabulary—Teacher's Reference

Coefficient: A multiplicative factor. A short form for repeated addition. You can also think of it as "how many groups." 4(3). 4 is a coefficient.

Factors: Point students out to the words **Fact**ory, and Manu**fact**ure. Numbers are made up of their multiplicative factors. Fact means to do or to make. You manufacture numbers with factors. They are the bricks!

Product: A factory makes a product! Factors make Products.

Prime: A prime number has exactly two factors—1 and itself.

Composite: A composite number has more than two possible factors.

The number 1: Is neither composite (because it has only one factor) nor prime (because it does not have exactly two factors).

Prime Factors: The factors of a number that are themselves prime. Every number has a unique set of prime factors.

Partial Products: When you break a number into parts and then multiply by those parts, the partial answers are the partial products. Ex: 43 x 10 = (40 + 3) x 10 → 400 and 30 are the partial products.

Exponents: An exponent is a short form to show repeated multiplication.

Materials List

<u>3rd 9 weeks</u>

- Base-ten blocks through 1,000*
- Fake Money through 1,000
- Journal Activities
- Work sheets
- 1x1 Tiles*
- Pop cubes*

*See Intermediate Math Kit from ETA recommended by C&I and generally available at your school.

Journal Activities

Week 19 Draw for me and then tell me how you could share 13 apples, without cutting any apples up. What does this tell us about the number 13?

Week 20 How do you know that 41x 5 is about 200 before you even work that problem all the way out?

Week 21 Draw a picture of $22 x 3 using ten and one dollar bills.

Week 22 Option 1 Look at the tiles on the floor. Each tile is a square foot. How many square feet of rug would it take to cover your whole classroom?

Week 22 Option 2 Draw a picture of a rug that is 9 square feet. Show how you know it is 9 square feet. Is 9 a perfect square? How do you know?

Week 23 Draw a 1x1x1 cube. How many cube units are in your cube? Draw a 2x2x2 cube. How many cube units are in your cube? Explain.

Week 24 In math, what are the three dimensions of an object? Why can't we draw something with 4 dimensions?

Week 25 Write a note to me describing where you need more practice before we assess.

Week 26 Can you write out what 10^{15} power is? Can you write out ANY power of ten? Why?

Week 27 Which number is biggest: 23x1000 or 10^{10} or 10 x 100 x 10000

Week 19: ReThinking Primes and Composites

Concepts:

Understanding Composite numbers as those that can be grouped using a **coefficient.**

Understanding Primes as those numbers that cannot be regrouped with a coefficient other than one or itself.

Materials:

Ones rods

OR Fake Money—Ones.

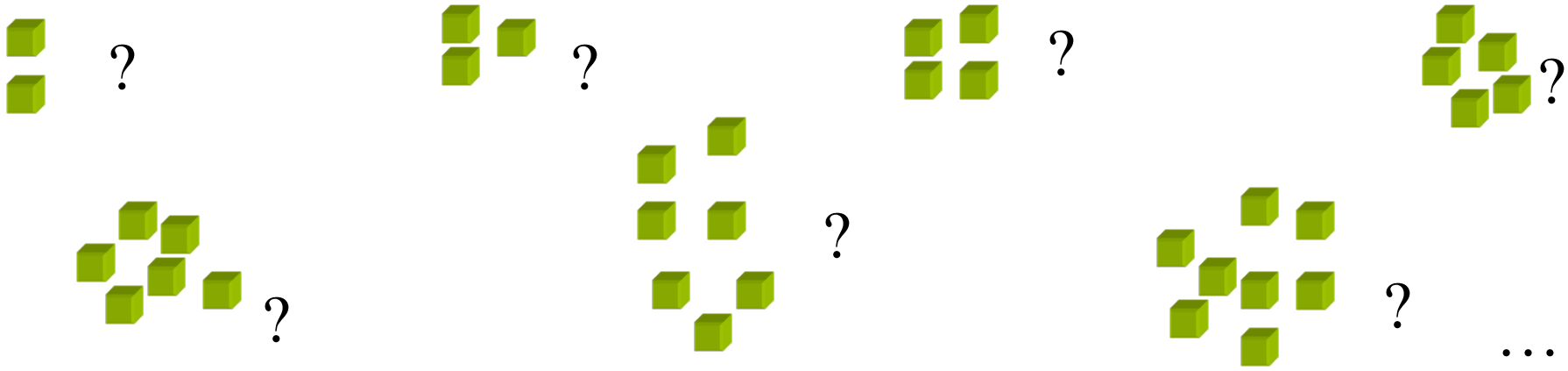
Lessons:

Students will explore prime numbers. This will be a review of SCoS work, but also will introduce to them the idea of multiplication as repeated addition and the idea of a Coefficient (the multiplier that essentially tells you how many times to add something up, or how many groups of something you have).

Week 19: ReThinking Primes and Composites--Notes

- Have the students play with and gain confidence with the idea that numbers can be written in different forms.
- Develop the idea that mathematicians find it helpful to know if a number can be broken down into more parts using whole numbers, or if it can not be divided up any more. They have named numbers either composite (can be broken down with whole numbers) or prime (cannot be broken down with whole numbers). Basically, can the number be divided up into some grouping of equal piles without using fractional parts?
- You can also think of this as: Can the number be written in the form of a multiplication problem [OTHER than 1 X the number]?

Week 19: ReThinking Primes and Composites—What it looks like



Let's go through the numbers from 1-13
<u>? = can this number be broken into any amount of equal piles</u>—other than all ones or the one whole group you started with?

37→ Can you break it up into equal piles and then write it in the form with a coefficient as a multiplication problem? If so, show it and write it. *(no)*

25→ Can you break it up into equal piles and then write it in the form with a coefficient as a multiplication problem? If so, show it and write it. *(yes 5(5))*

19→ Can you break it up into equal piles and then write it in the form with a coefficient as a multiplication problem? If so, show it and write it. *(no)*

Week 19: ReThinking Primes and Composites—example dialogue

*We're going to look at Prime and Composite numbers again. This time, we'll think about the **form** of the number. Basically, a composite number is one that you can rewrite as a multiplication problem using whole numbers other than 1 X the number and a prime is not. A prime can be written as a multiplication problem, BUT not unless you use either one or a fraction: (1 x 3 = 3; 1 ½ x 2 = 3).*

A composite number can be written as a multiplication problem, it can be written in multiplication form using a whole number coefficient other than 1. For example, 4 can be written as 2(2). That means 2 two times. Or you can think of it as "2 groups of 2"

Another way to think about it is that composite numbers can be broken up and shared without using fractions. You only need one "counter-example" to <u>prove</u> that a number IS NOT PRIME. If a number is not prime than it is composite (except for the number 1 which is neither composite of prime!)

We're going to be using ONES this week, because we are looking at how numbers can be grouped with <u>whole numbers.</u> A one is a whole. It is the basic unit and will help us to see how the numbers can be grouped using WHOLES.

Week 19: ReThinking Primes and Composites—example dialogue

*Let's say you have 20 dollars. You could share that several different ways using whole numbers. You could have 2 groups of (10) or 10(2) or 5(4) or 4(5). If you can share it <u>any other way</u> besides 1(20) or 20(1), then it's considered C**omposite.***

Try 17 with 17 wholes—17 one dollar bills.

Get with a partner and try 36, then 37, then 38 and then 39. Use all one dollar bills. See if you can share them some way other than giving every one 1 dollar bill or having one person keep it all. Write down the different ways you can evenly share each number and determine if any of those numbers are prime.

*A neat word to know is the word **Coefficient.** Coefficient just means a multiplier. As we know, that just means how many times you add up a certain number to see the value in a different form. For instance if I break 12 down into 4(3) I can say that 4 is the coefficient and I add up 3 four times. Or I could write it as 3(4) and call three the coefficient which means to add up 4 three times.*

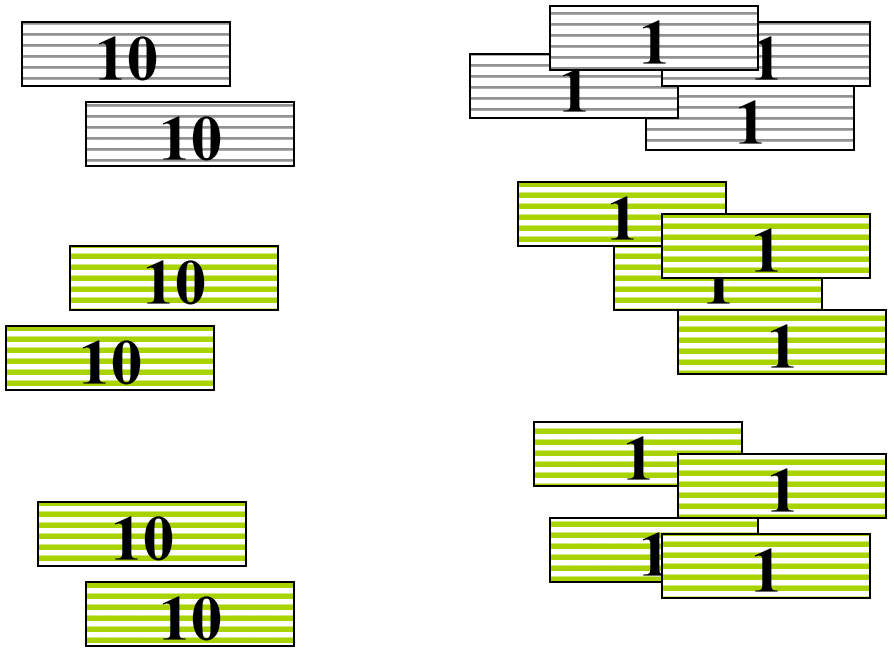
Is it Prime or is it Composite?

Number	**Drawing**—is there a counter-example to prove the number is not prime?	P or C
21		
23		
28		
29		

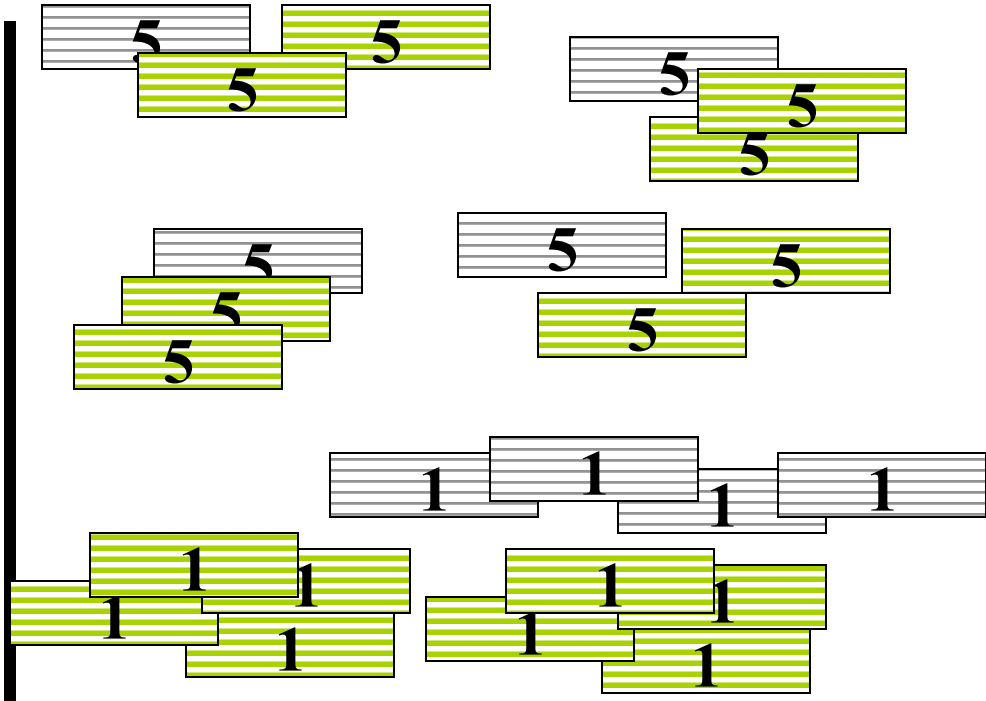
Is it Prime or is it Composite?

Number	**Drawing**—is there a counter-example to prove the number is not prime?	P or C

Week 21: Playing with Multiplication pt. II—what it looks like



60 + 12



15 + 15 + 15 + 15 + 4 + 4+ 4

24 x 3 (20 + 4) 3

24 + 24 + 24

20*3 + 4*3

20 +20 + 20 + 4 + 4 + 4 =

60 + 12 = 72

Standard Form

24x3 = (5+5+5+5+4)3

5*3 + 5*3 + 5*3 + 5*3 + 4*3 =

15 + 15 + 15 + 15 + 12

30 + 30 + 12 =

60 + 12 = 72

Week 22: Squares: A special exponent!

Concepts:

- Just as Multiplication is repeated addition, exponents are repeated multiplication
- Develop Idea of "to the 2^{nd} power"– Squares

Strategies:

Develop students' abilities to "see" a squared number.

<u>Materials:</u>

1x1 square tiles

Lessons:

Develop concept of exponents beginning with the concrete concept of squares. To the second power can be seen both algebraically and geometrically. Combine these two concepts for students this week.

Exponents mean repeated multiplication

The to the second power is a special exponent which we can represent in the physical world.

Week 22: Squares: A special exponent!--notes

- Explain to students that exponents are to multiplication as coefficients were to addition—exponents tell you how many times to MULTIPLY a number by itself. Just as a coefficient told you how many times to add that number up, exponents tell you how many times to multiply it out, beyond that number itself.
- Show students that 5^2 means 5x5 = 25.
- Show students why we call that a Squared number—a number to the 2^{nd} power can be represented in the physical world. It is "2-dimensional" and tells us how much 2 dimensional space is taken up. We call these "square units" and it is what we mean by "Area".
- Make sure to use non-examples a few times this week—ask what 2(5) means to make sure they are noticing the difference between exponents and coefficients.

Week 22: Squares: A special exponent!—what it looks like

$5^2 = 5x5$ 

$4^2 = 4x4$ 

Yikes! Can you do Square Roots?
If you have a big square made of 36 little squares, How long is one linear side?

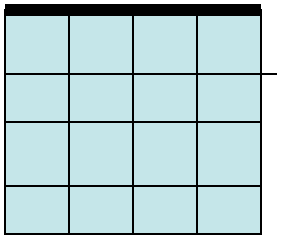
What is 4^2

It means

'make a square out of your 4 unit linear side' 

--4 linear units--

1
1
1
1



You'd get how many little 1 by 1 inch squares? $4^2 = 16$

The one side you started with is called the Base or the Root. It is linear; it is the length of one side.

Think of your answer as a Square made up of a bunch of little squares.

Week 22: Squares: A special exponent!—**Example Dialogue**

*What does 2(5) mean? Right, it means 5 **+** 5 or add your 5 up two times. So an equivalent form of 2(5) is 5 + 5 or 10, correct? Good. Remember that the coefficient tells you how many times to add something up. An exponent is written like this 5^2 and means, multiply out your number that many times. So 5^2 means 5 **X** 5.*

*5^2 means 5 **X** 5 = 25. 4^2 means 4 **X** 4 = 16. What do you think 7^2 means? Good, means 7X7 which is 49.*

There are a LOT of different math terms associated with exponents. We can say 5^2 many different ways:

5 raised to the 2^{nd} power, 5 to the 2^{nd} power, 5 to the power of 2, and 5 squared.

Let's see why mathematicians call a number to the 2^{nd} power a SQUARE. Does that mean that it's a boring number? No! It really has to do with Geometry and the shape of a square.

What do you remember or know about Squares? What do they look like? What do you know about their sides? Remember a Square is just a special rectangle that has all of the sides the same length. This is important. If we know one side of the square, we know all about that square! Look, I'll draw a four inch line which we can write as 4 or 4^1. If that is one side of the square, can you draw the rest of that square? Do I have to tell you how long the other sides are or do you already know? (see what it looks like).

Week 22 Squares: A special exponent!**—Example Dialogue**

What if I drew a 4 inch side and told you to draw a rectangle from it. Do you see that you could draw that rectangle many different ways? (show a 4x5 rectangle and other examples). But, if I tell you the shape is going to be a ***square****, we KNOW all of the sides!*

Now, let's see if we can measure that square. One way that we measure squares, and all flat things, is with square units. Look here (see "--what it looks like"). Here is my 4 linear units, it could be inches or feet or centimeters or miles, or anything. Let's use inches. If we "make a square" out of that line, we end up with a square like this. To find the AREA of that square we can multiply 4x4 we can which is the same as 4^2.

Let's look at squares further. Take out 4 square inch tiles. Line four of them up. Do you see that one side is 4 inches? Now, make a square that is 4 inches on each side with your 4 inch tiles. (Some students may struggle with this, guide them until they get the hang of it). Good. Now count up your total squares. What did you get? Now put 4x4 in the calculator, then put 4^2 *in your calculator. What does this all mean. What is* 4^2 *? How many 1 inch squares are in your 4X4 square?*

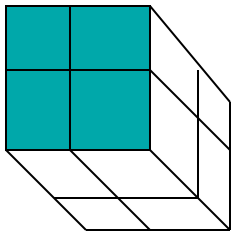
Let's try it with a 3X3 square. Now, work with a partner and show me what 7^2 *looks like as a square and show me what* 7^2 *equals.*

Build a square with 36 little squares. Now tell me how long is one side. You've just done a "square root"! You've found out how long a side is when you knew the area of the whole square!

Try and build a square with 10 tiles. Can you do it? No—10 cannot be written as a power of 2. Can you make a square with 12 tiles? Can you make a square with 64 tiles? Can 12 or 64 be written as a power of 2?

Week 23: Cubes: Another Special Exponent!—What it looks like

2^3



Use pop cubes to build different larger cubes and explore the answers.

Yikes! Can you do CUBE Roots?
If you have a big cube made of 27 little cubes, how long is one linear side?

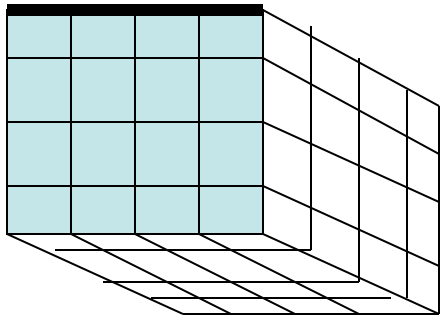
What is 4^3 ?

It means

'make a CUBE out of your 4 unit linear side'

--4 linear units--

1
1
1
1



You'd get how many little 1 by 1 inch cubes? $4^3 = 4x4x4=64$

The one side you started with is called the Base or the Root. It is linear; it is the length of one side.Think of your answer as a Cube made up of a bunch of little cubes.

Week 25: Powers of 10!

Concepts:

- Just as Multiplication is repeated addition, exponents are repeated multiplication
- Develop fluency with Powers of 10.

Strategies:

Develop students ability to see the magnitude of a power of ten very quickly

Materials:

Worksheets

Money—10s, 100s, 1000s

Base-ten blocks through 1000

Lessons:

Develop idea that powers of ten are special because our number system is Base-Ten.

Exponents mean repeated multiplication

Teach students to multiply by tens quickly.

Making Connections between multiples of tens and squares and cubes.

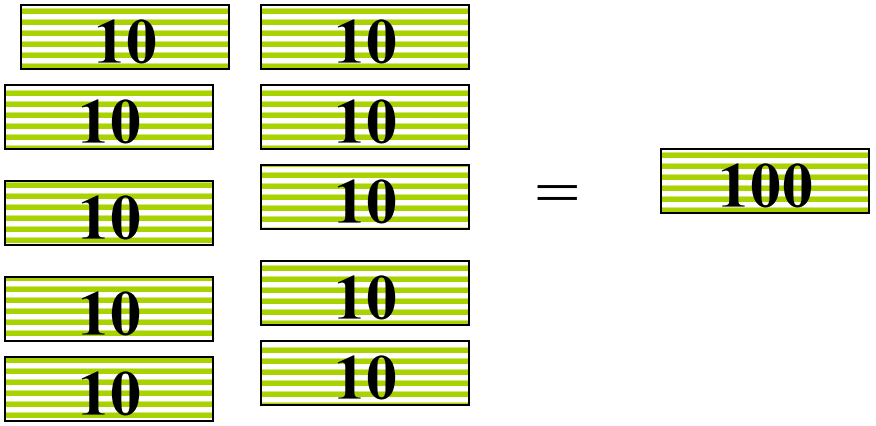
Week 25: Powers of 10!--notes

- Powers of ten are special because our number system is in base ten. When you multiply by ten you are always regrouping to the next place value.
- Scientists and mathematicians use powers of ten to think about numbers because it tells them quickly about how big a number is.
- Use Base ten blocks to review and make connections between powers of ten and linear, squared and cubed measurements.
- Use the money to show how powers of ten regroup every time because it always gives you a set of ten. Do other powers do that?
- Compare any other power set to the powers of ten for the students.
  - Ex: 8^2, 8^3, 8^4, is 64, 512, 4096 **vs.** 10^2, 10^3, 10^4, is 100, 1000, 10000
  - Why is the one so messy and the other so NEAT?

–Teacher Note: The ones cube can be written as a power of ten: $10^0 = 1$. Anything to the zero power is equal to one BECAUSE it implies an equal amount of 10s in the numerator and denominator of the number. Think of 10^2, for instance as being (10x10x10x10)/(10x10). When we factor out the 10/10 = 1s in that, we end up with 2 powers of ten in the numerator or 10^2. When we have (10x10)/(10x10) for instance, when we factor out the 10/10=1s we end up with 1/1 and no tens in the numerator or denominator or 10^0. If you read this and don't understand, email me for further discussion. The kids don't need to know this yet, but I want you to understand it so you are more likely to say mathematically accurate things around these discussions. Make sense?

Week 25 Powers of Ten—what it looks like

Exponential Form	Long Form	Simplified Form
10^1		
	10x10	
		10,000,000



10(80)

23(10)

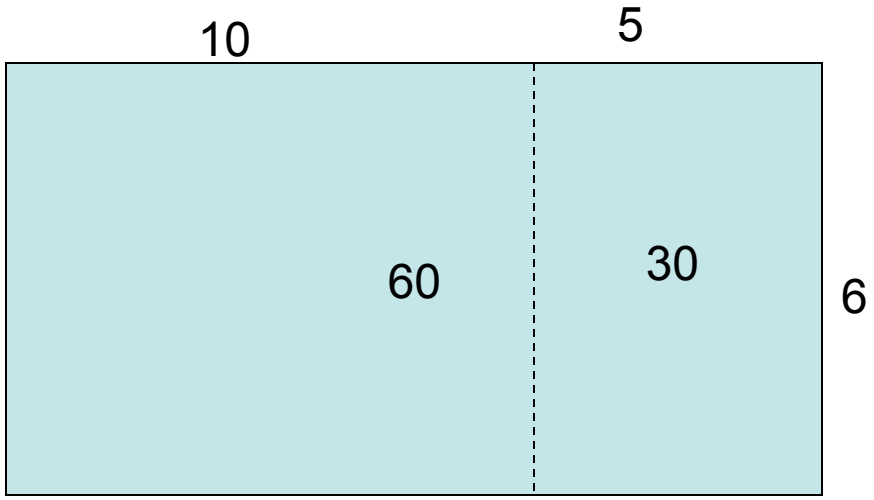
32(100)

If I make 90 dollars a week for ten weeks, how much money will I have made by the end of the ten weeks?

420(10)

Distributive Property and Quadratics
The Rectangle Sequence

First: Develop understanding with 1x2 digits with whole numbers



Squares are pre-divided up for Students. Students focus on The relationships only.

Here is an example of 15 x 6.

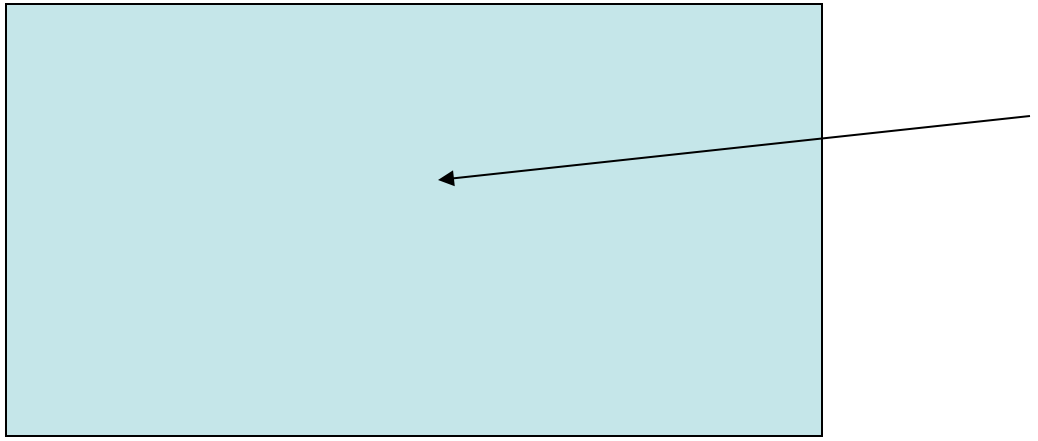
Student will divide 15 into 10 and 5 and then complete problem as shown here.

15 * 6 = (____ + ____) 6 = _____ + _____ = _____

15 * 6 = (__10__ + __5__) 6 = __60__ + __30__ = ___90__

To be clear: This is what student sees:

Multiply 15 * 6 using the distributive property and base-ten to make the problem easier

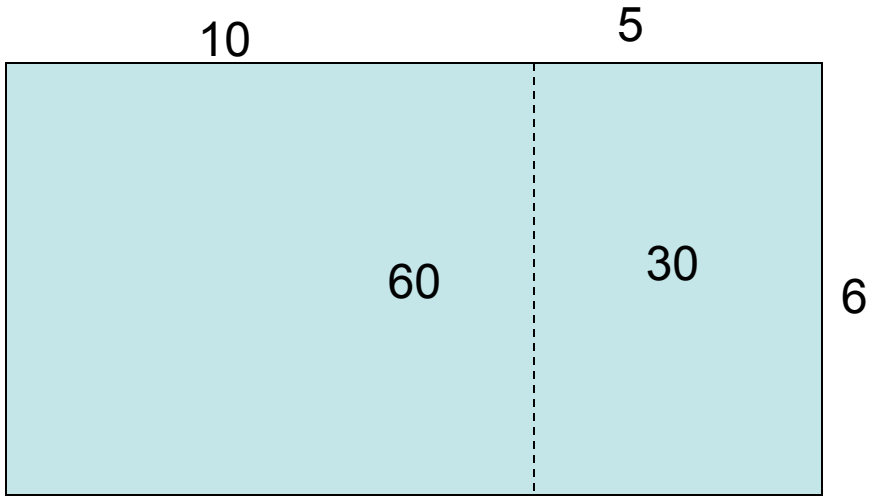


Squares are pre-divided up for Students. I did not cut this into 15 * 6, but it should be cut up into Exactly 15 * 6 (not a big grid that student has to cut up first).

.

15 * 6 = (____ + ____) 6 = _____ + _____ = _____

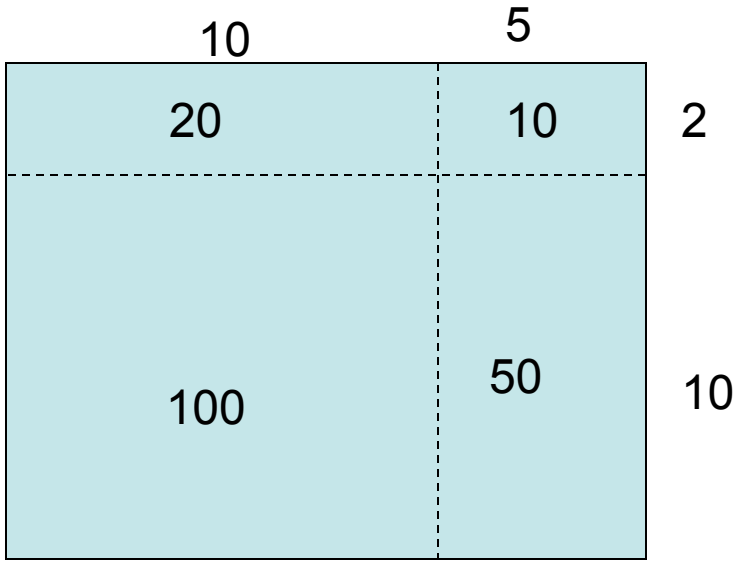
And this is what student does to complete the problem



15 * 6 = (__10__ + __5__) 6 = __60__ + __30__ = ___90__

The Sequence goes next to 2x2 with whole numbers

Second: Extend understanding with 2x2 digits with whole numbers



Squares are pre-divided up for Students. Students focus on The relationships only.

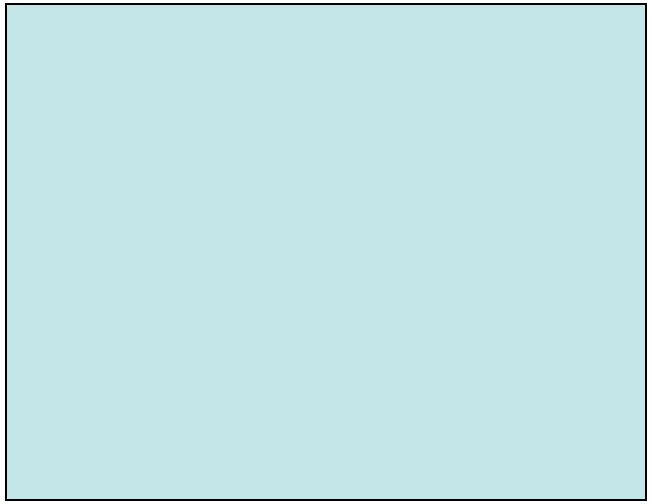
Here is an example of 15 x 12.

Student will divide 15 into 10 and 5 and 12 into 10 and 2

15 * 12= (____ + ____) (_____ + _____) = _____ + _____ + _____ + _____= _____

Again: this is what student sees:

Solve 15 x 12 using what you know about the distributive property and base-ten

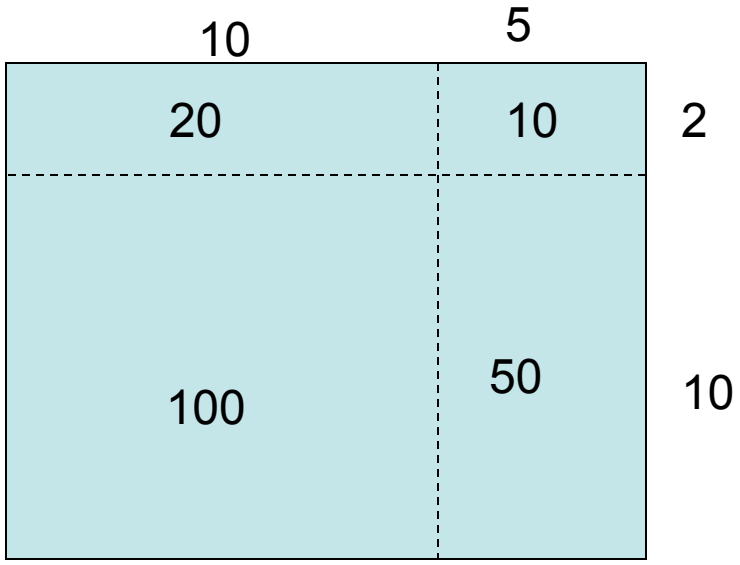


Note: rectangles are pre-scribed for Students. This would be a 15 x 12 (180 little squares) rectangle

So Students focus on The relationships only.

15 * 12= (____ + ____) (_____ + _____) = _____ + _____ + _____ + _____= _____

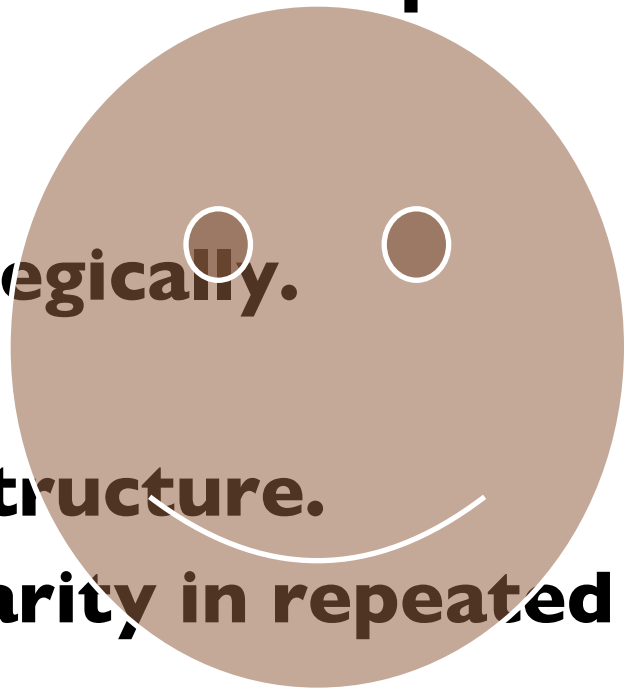
And this is what the student does:



15 * 12= (_10__ + _5__) (_10___ + __2__) = __100_ + _50__ + _20__ + _10__= _180__

Common Core Practice Standards

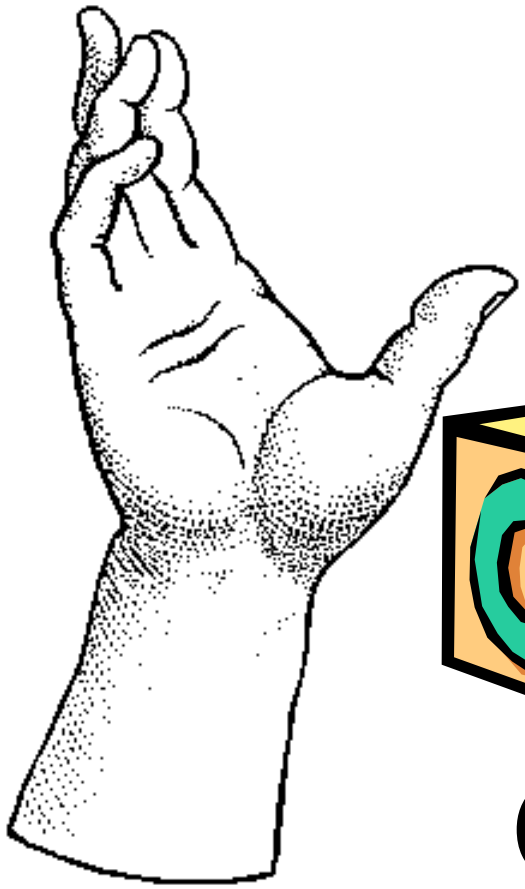
1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

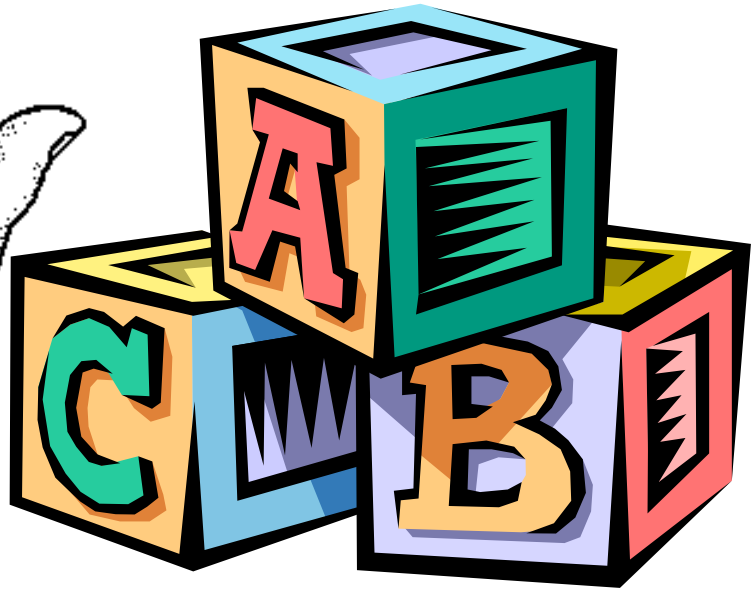


Classroom

Discuss with your partner/group how you will set up your class this year to incorporate the Warm-ups and Standard Course of Study

What further questions do you have and what will you need to work on?





Quarter 4

Set 1 Resource Math

Remedial Warm-ups & Assessment

9 Week Division Outline

Intro to Division	Division is Related to Subtraction	Division is Related to Multiplication
Quotients and Fractions	More Connections with Division	Division in Life!
Division in Life! Part 2	What have we learned this year?	Years' End Evaluation

Quarter 4: Division

We will approach division as repeated subtraction and make the connection to multiplication. This will prepare students to understand division of fractions, and to have a way to solve problems even if they struggle with the traditional algorithms.
We will also think of division as essentially a ratio. We want students to see the equivalent forms of division.
We will not emphasize remainders or long division. We, instead, will put energy into the language of division and understanding what division is.
We also want the students to be able to estimate answers.
In this quarter we will cover:

- understanding division: related to subtraction and multiplication
- distinguishing between division and multiplication in word problems
- understanding division language
- seeing the equivalent forms of a number.

Implementation Notes:

Review the teacher's vocabulary reference and the definition of division. Note that we do not say that the Dividend is "the big number." This is not mathematically correct. When we divide by fractions our quotient has a larger value than our dividend by itself: 3/(1/2) = 6.
Do not speak incorrectly and call the dividend the big number. Use the language in the teacher's reference to make sure you are mathematically accurate.

Vocabulary—Teacher's Reference

Dividend—the amount being divided. It is a quantity that is being divided by another quantity. It is essentially a product of two factors (the divisor and the quotient).

Divisor—the number of shares by which you divide the dividend. The divisor is also known as a factor.

Quotient—the value of a given ratio; the total amount of each share; the result of a division problem. The quotient is also known as a factor.

Division—taking the ratio (x/y or $x \div y$) of two numbers x and y.

CRC Concise Encyclopedia of Mathematics, 2nd ed.
Eric M. Weisstein.
Definition of Division

This definition helps us to see some connections that we don't always make and about which we are often not precise.

Division: *Taking the RATIO x/y of two numbers x and y, also written x ÷ y. Here, x is called the DIVIDEND, y is called the DIVISOR, and x/y is called a QUOTIENT…If left unevaluated, x/y is called a FRACTION, with x known as the NUMERATOR and y known as the DENOMINATOR…*

…Division is the inverse operation of MULTIPLICATION, so that if a x b= c, then a can be recovered as a = c ÷ b as long as b ≠ 0. In general, DIVISION BY ZERO is not defined since the ability to "invert" a x b = c to recover a breaks down if b = 0 (in which case c is always 0 independent of a).

Cutting or separating an object into two or more parts is also called division.

Note—this was also in the definition, but is relatively obscure information:

The symbol "/" is called a SOLIDUS (or DIAGONAL), and the symbol "÷ " is called the OBELUS.

Discussion of the Definition

Let's notice something really, really important: Do you see that it says that the quotient is defined as "x/y"? Didn't you expect them to say that the quotient was some other number, z. Some number that was the "answer" to "x/y"? I know I did. And then I realized that this is a remnant of how I was taught. We were not really taught to think about EQUIVALENCE deeply and notice equivalence in such a way that we immediately internalize and see it. That is to say that we are used to saying that for x/y = z that x is the dividend, y is the divisor and z is the quotient. We think of them as separate names, separate entities: definitions to memorize rather than as RELATIONSHIPS. They are, essentially, relationships.

Now, notice that x/y IS z. They are the same value, so we can say that z is the quotient if we are given the information that z is equal to x/y. But we can also, more precisely, say that x/y is itself the quotient. Wacky, huh? We were taught to think of the quotient as something separate from the x/y rather than thinking of it as that value or relationship, so we think to define it according to where it is, rather than as that essential ratio. Does that make sense? This underscores our tendency to look for an "answer" rather than to explore the relationships of numbers. I could say that z is the quotient, but it's more valuable to see that the relationship, or ratio, x/y is itself the quotient. We can then evaluate this number and write it in a different FORM if we want to. This different form could be z, but it's that ratio itself that DEFINES the quotient.

Lastly, notice that the definition states that "if left unevaluated, x/y is called a fraction." Again, kids (as a product of our instruction) do not see this connection and think of fractions as a totally different type of problem. Working with fractions is working with unevaluated division problems or ratios. As mathematicians, the kids need to learn to

Materials List

<u>4th 9 weeks</u>

- Base-ten blocks through 1,000*
- Fake Money through 1,000
- Journal Activities
- Work sheets
- A bowl of 6 cups of beans and 12 clear cups (preferably 8 oz cups)

*See Intermediate Math Kit from ETA recommended by C&I and generally available at your school.

Journal Activities

Week 28– 6 ÷3 = 2. Label the parts of this problem and explain what is happening in a division problem.

Week 29– How is division related to subtraction?

Week 30– How is division related to multiplication?

Week 31– Terry and Barney are arguing about the value 10/2. Barney said, "that's a division problem". Terry said, "no, that's a fraction". Who is right?

Week 32– Would you rather have 10 dollars to share with 2 people or 2 dollars to share with 10 people? Why?

Week 33– Create a division problem that involves money. Write it out, draw a picture of it, and solve it.

Week 34– Create a division problem that involves something other than money. Write it out, draw a picture of it, and solve it.

Week 35– Tell me about the math concept that you are the proudest of learning this year. What is it, and why are you excited about it?

Week 36– Tell me what you want to work on more and understand better, of all that we have done this year.

Week 28: Intro to Division

Concepts:

Division is one way to make sense of, or simplify, a ratio.

The language of division: Dividend, Divisor, Quotient.

Materials:

Fake Money: 100s, 10s, 1s

Lessons:

Develop the language of division and the concepts that the language communicates.

Use basic problems to develop language.

Week 28: Intro to Division--Notes

- You will be working to Undo some misunderstandings about division that students have learned from teachers and others. They think division is all about making things smaller. You need to show them that Division is a ratio, or relationship, that can be expressed in different forms.
- You need them to see that a Quotient is not "the answer." The quotient is that relationship (x/y) which can be written in different forms. We can leave it in fractional form or we can look into it more and see if we like it better in a simplified form. **If you are told that x/y = z then both x/y AND z are the quotient because (x/y) is EQUAL to z. Make sure you understand this.**
- Watch yourself and make sure that you do not say incorrect mathematical statements.
- Remember that teaching the vocabulary means using it precisely and repeatedly. Develop this language in your classroom in a non-threatening way. The words sound a lot alike and may confuse the kids. Use the words to help them develop the concepts and vice versa. It's most important that they see the relationships and that they then learn to use those words to more effectively communicate their understanding of these concepts.

Week 28: Intro to Division—What it looks like

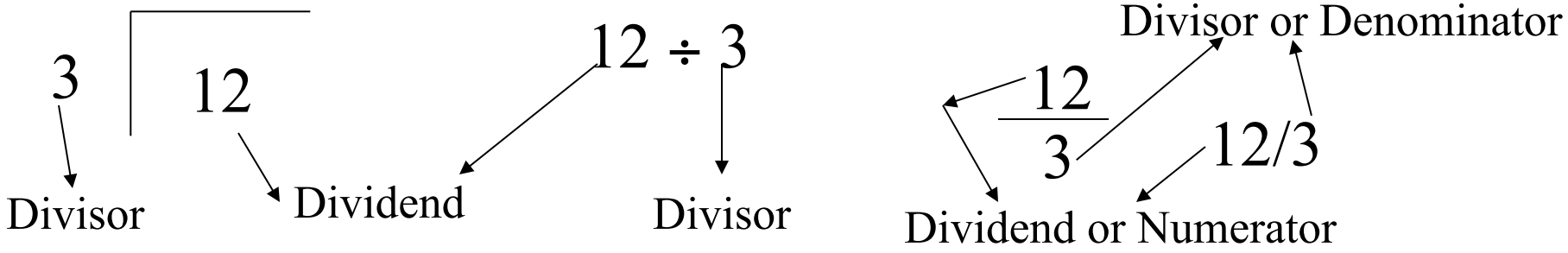
$3\overline{)12}$ $\quad 12 \div 3 \quad \frac{12}{3} \quad 12/3$

$4\overline{)1}$ $\quad 1 \div 4 \quad \frac{1}{4} \quad 1/4$

$10\overline{)100}$ $\quad 100 \div 10 \quad \frac{100}{10} \quad 100/10$

Week 28: Intro to Division—example dialogue

Okay let's learn all of these words so that we are all speaking the same language. The "top dog" is the Dividend. No matter what form we write it in, it's still the dividend. What do we usually call the dividend when it's in Fraction form? Yes, remember, it's called the Numerator.

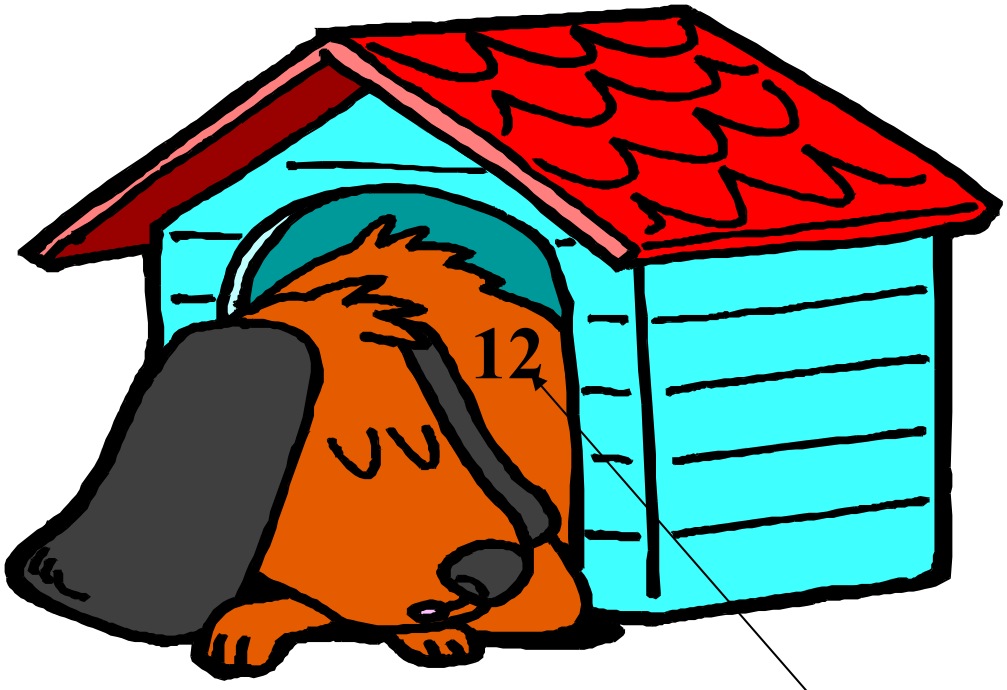


Now the other part of the ratio is called the "Divisor." It's what we divide by. Again, in fraction form this part of the ratio is also called what? The Denominator.

The whole thing is the Quotient. The whole thing is a certain value, and that value is the Quotient. We can write the quotient in a simplified form if we want to, but it's still the same value, if we have evaluated it correctly. An un-simplified quotient is also called a fraction. For instance our quotient here is what? 12 ÷ 3. Good. But we can look at that relationship and simplify it. Some of you might have that memorized as a fact: does anyone know what 12 ÷ 3 is in simplified form? 4 good. We could also have figured that out with our money. This would mean, for instance that we had a Dividend of 12 dollars and needed to share between 3 people because our Divisor is 3. Let's do that out the long way with our money and see how much each person would get… 12 ÷ 3 means that each person would get 4, or 12 ÷ 3 equals 4. It's the same value, so you can write it either way, depending on what works for you as the mathematician. They are both "the quotient."

The Top Dog gets the Dog House

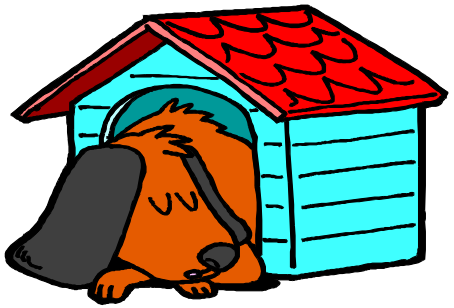
$12 \div 3$ $\qquad \frac{12}{3} \qquad$ $12/3$



Inside the "dog house"

$3\overline{)12}$

Converting to different forms: division



25/5

25 ÷ 5

$\overline{)\,5\quad 25}$

¼

---

3 ÷ 8

$4\overline{)\,1}$

$1\overline{)\,4}$

Week 29: Division is Related to Subtraction

Concepts:

Division is repeated subtraction

We can use repeated subtraction to solve division problems.

Materials:

Fake money 100s, 10s , 1s

And/or 1s, 10s and 100s units.

~~A bowl of 6 cups of beans~~.

Lessons:

Once we understand the idea that a quotient is a relationship, we need to then have strategies for figuring out how to simplify that relationship into other forms.

It's important to see the connection between different operations.

Math is not "procedures," Math is relationships.

Week 29: Division is Related to Subtraction--Notes

If we are given a quotient, how can we think about that quotient so we can simplify it, or write it in a different form?

One way to think about division is to realize that it is just repeated subtraction, just as multiplication is repeated addition. You can think about word problems and numbers, and solve division problems by using repeated subtraction.

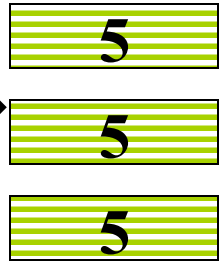
Make sure the students see this connection and can talk about it, explain it and show it with the manipulatives you use (money or the 1s and 10s rods).

Develop students' ability to turn a number problem into a word problem to reinforce their understanding.

Your kids will understand more about division (including the identity properties, dividing by fractions and remainders) if you keep the examples fairly contained. See "what it looks like". Do one Dividend with different Divisors for a whole day before moving to a different Dividend. By the end of week try keeping the same Divisor and switching the Dividend. Have students discuss this.

Week 29: Division is Related to Subtraction—What it looks like

15/5 →

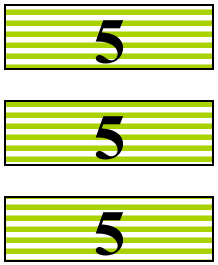


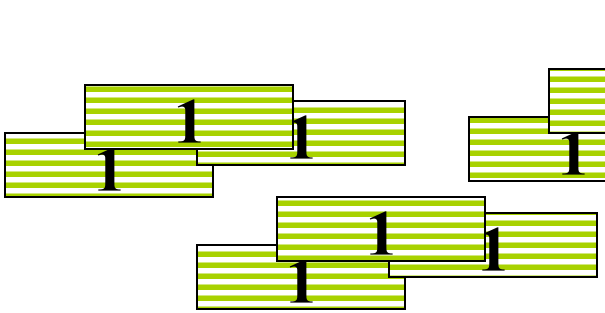
1 pile of five

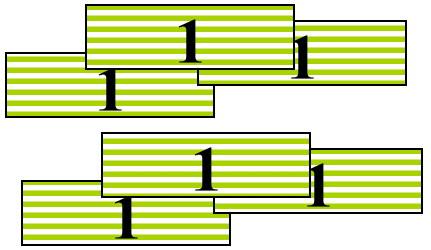
2 piles of five

3 piles of five

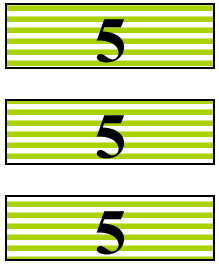
15/3→



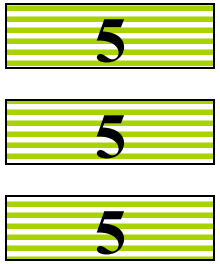




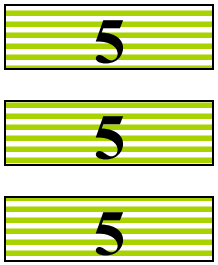
15/1→



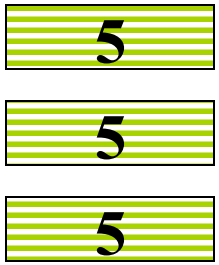
15/15→



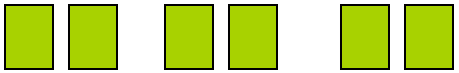
15/(1/2)→



15/2→



Review with beans to solidify concepts:



= 6 total cups

2 x 3 = 6 = (2+2+2)

6 cups

If I have 6 cups of beans and give 2 cup portions, how many portions can I serve?

6 ÷ 2 = 3

Week 30: Division is related to Multiplication—what it looks like

$10 \times 3 = 30$

$10 + 10 + 10$

3 piles of ten

$30 \div 10 = 3$

How many tens can I subtract out of 30?

$3 \times 4 = 12$

$12 \div 3 = 4$

$10 \times 10 = 100$ use money or units

$100 \div 10 = 10$ use money or units

Play with this idea back and forth: Notice the different ways you can invert the numbers.

$8 \div 2 = 4$ $4 \times 2 = 8$ $8 \div 4 = 2$ $2 \times 4 = 8$

Etc.

$4 \div 0 = ?$ Does anything by 0 get us our 4 back?

Week 31: Quotients and Fractions

Concepts:

Fractions and Quotients and Decimals are all related. They are different forms of a value.

<u>Materials:</u>

Fake money 1s, 10s, 100s, quarters, nickels and pennies.

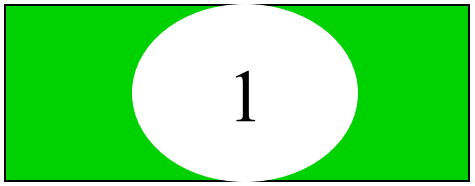
Lessons:

Have students play with different ways to see numbers.

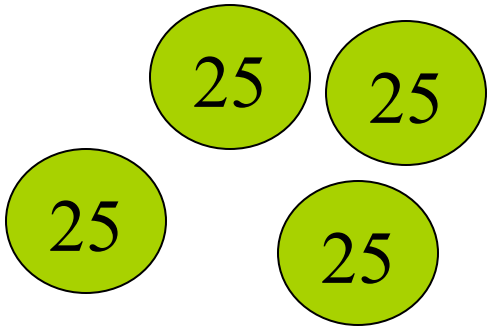
They must see the connections between these different systems.

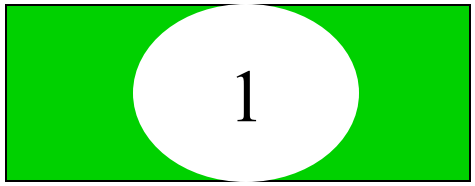
The Mathematician can see numbers in different ways and uses the Form of the Number that works for them in a given situation.

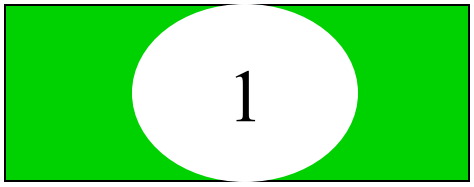
Week 31: Quotients and Fractions—what it looks like

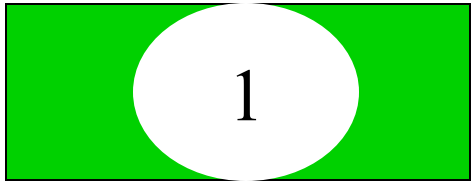


What if you had one dollar and had to split it between 4 people? Your Dividend is 1 and your divisor is 4.

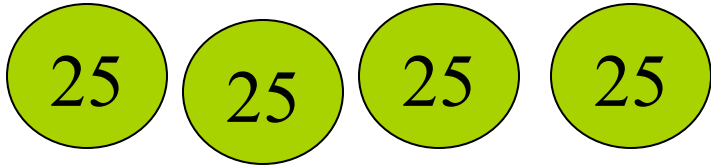


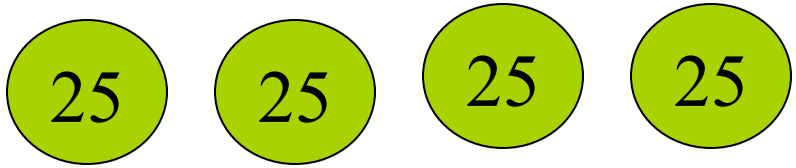


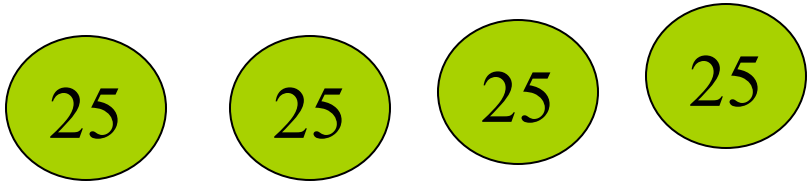




What if you had three dollars and split it between 4 people?

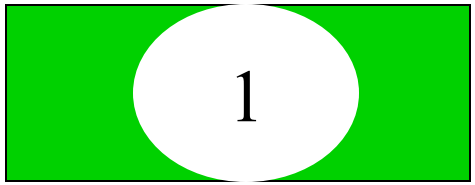


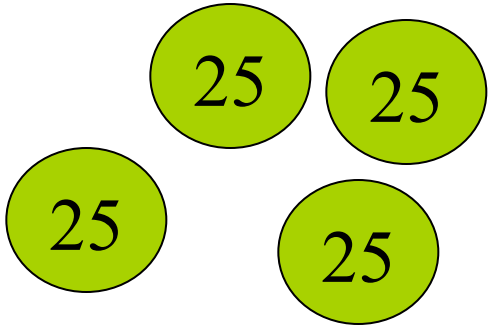




Week 32: More Connections with Division—what it looks like

Worksheets
For week 32

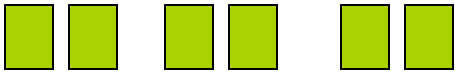




$12 \div 3 = 4$

If needed

Review with beans to solidify concepts:


= 6 total cups

$2 \times 3 = 6 = (2+2+2)$

If I have 6 cups of beans and give 2 cup portions, how many portions can I serve?

$6 \div 2 = 3$

Week 33: Division in Life: Part 1

Concepts:

Use different scenarios to see how division operates in life.

Strategies:

<u>Materials:</u>

Fake Money

Lessons:

Develop students' ability to work through the meaning of word problems.

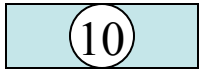
Distinguish between a division problem and some other type of problem by drawing out what's happening or otherwise make sense of the language.

Week 33: Division in Life: Part 1--notes

- Have students help you develop word problems as the week goes on.
- Make sure students are drawing pictures, or otherwise making sense of the problems. Use some non-examples to ensure that they are not just "doing division" because that's what you've been "doing" lately. Mix it up a little so that they have to talk through each problem.
- Emphasis sharing type of division problems. This would include portion sizes and money as well.
- Work to teach the kids some life skill math basics, particularly: days in a week and days in a work week.

Week 33: Division in Life: Part 1—what it looks like

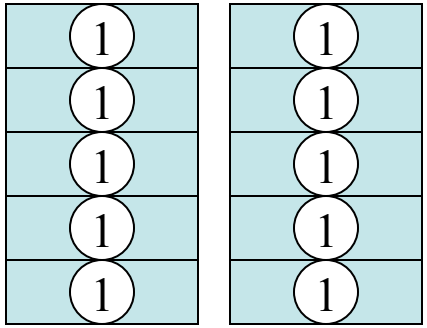
If you and a friend found 10 dollars and wanted to share it equally how much would each of you get? Okay—let's think about this problem first: would you each get less than one, more than one or exactly one dollar? Why?
Have kids follow a basic three step approach to each problem:
1) Set-it up/draw it 2) Decide what need to do 3) Solve it



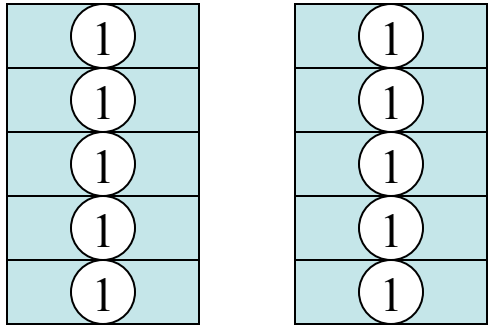
You've got 10 dollars

And you need to divide it by 2

10÷2



Break it up into ones



Each person gets 5 dollars

10÷2 = 5

Note that a student could solve this through repeated subtraction as well. They would have 5 piles of 2. Each pile would represent one dollar for each person , five times.

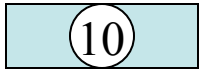
Throw in a non-example!: If you earned 10 dollars and your friend earned 10 dollars, how much did the 2 of you earn altogether? *solved with addition or "rapid addition"/multiplication because it's the same number 2 times.*

Week 34: Division in Life: Part 2—what it looks like

Let's say you found 10 dollars and you were by yourself; there was no way to know who it belonged to so you're going to keep it. Okay—let's think about this problem first: How would you set this up as a division problem. What do you expect your quotient to be? Why?

Have kids follow a basic three step approach to each problem:

1) Set-it up/draw it 2) Decide what need to do 3) Solve it



You've got 10 dollars

And you are keeping it by yourself

$10 \div 1$

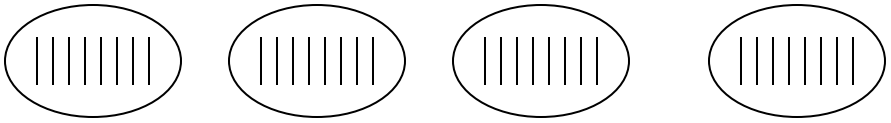
Discuss the identity property with students

Note that a student could set this up as a subtraction problem as well (10-0 = 10) Validate that decision as well, and then reinforce writing it as a division problem.

Throw in a non-example!: If you earned 10 dollars a day for a week, how much did you earn altogether? *solved with addition or "rapid addition"/ multiplication because it's the same number 7 times.*

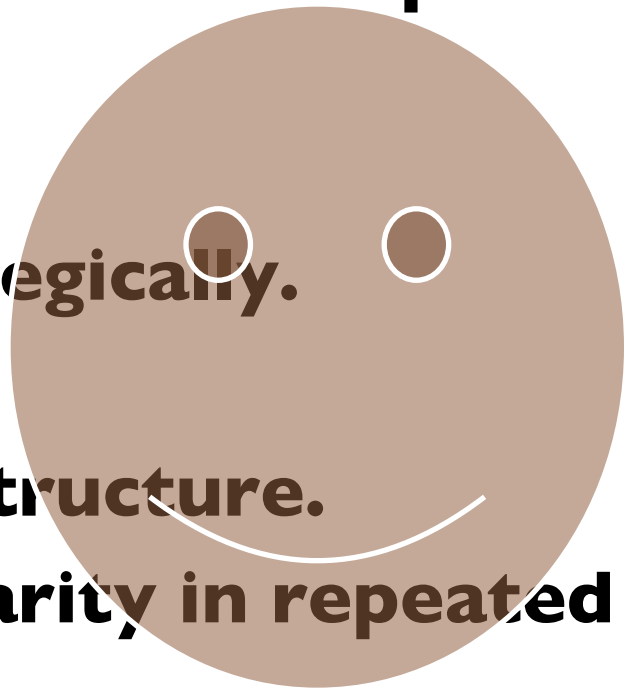
Week 34: Division in Life: Part 2—example dialogue

Okay let's take a look at what division is. Put out a Dividend of 32 on your table. Let's divide it by different Divisors and see what happens. Help me make story problems for the different math expressions, okay? I'll do the first one. I earned 32 dollars last week and I worked on 4 different days. I earned the same amount each day. How much did I earn each day? First of all, did we earn more than 1, less than 1 or exactly 1 dollar per day? How do you know? How would you set up this problem symbolically? Now, draw a picture to show me how to solve this problem. Ex:



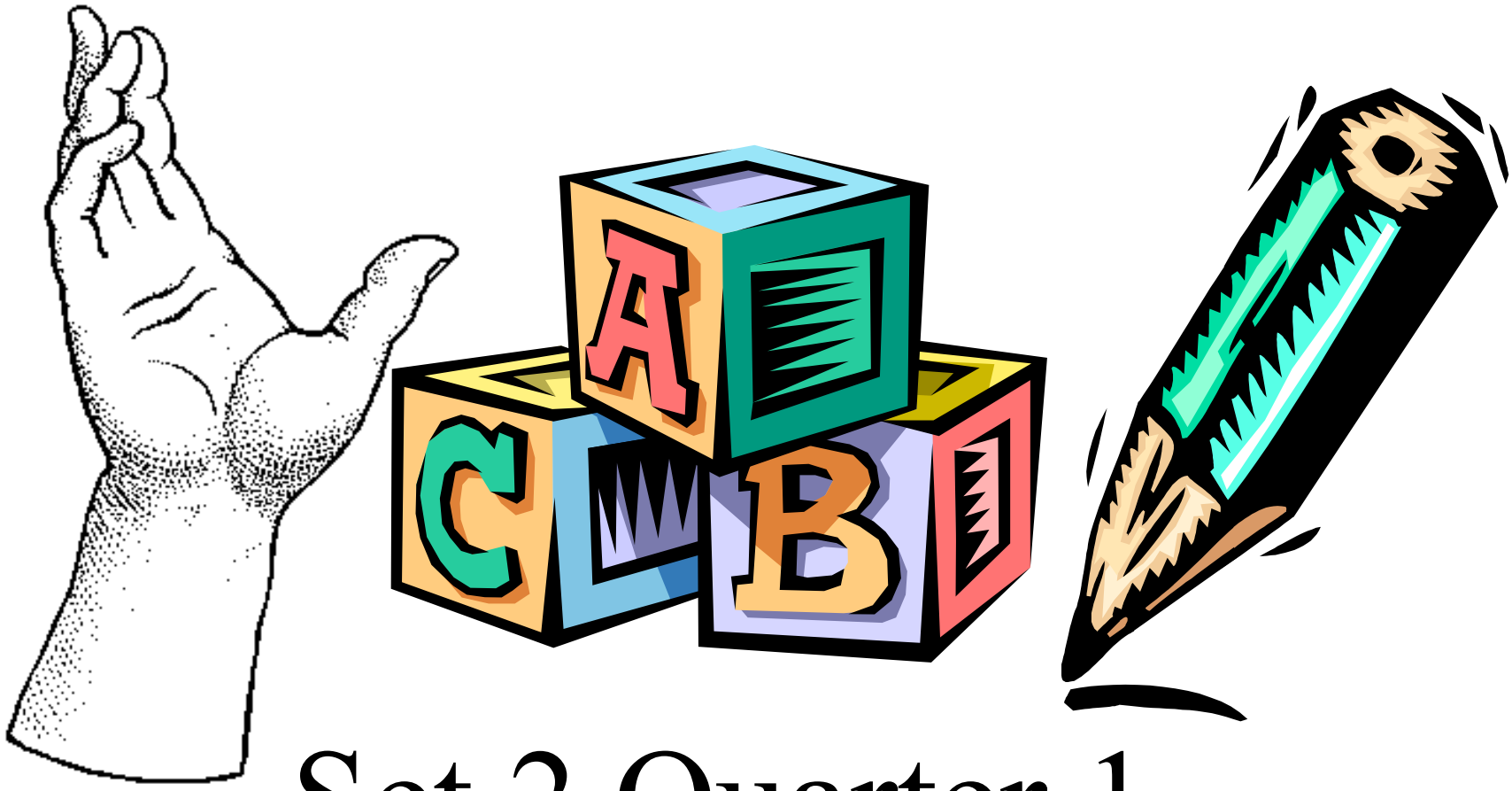
Common Core Practice Standards

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**



Classroom

Make a list with your group of language issues you will need to watch with yourself as you discuss math with your students.



Set 2 Quarter 1

WCPSS Middle School Remedial Warm-ups

Wake County Public School System/Valerie Faulkner vfaulkner1@wcpss.net

Two stories

- You have a dime (1/10 of a dollar) and seven nickels (7/20 of a dollar).
  - In fractional form show how much money you have all together.

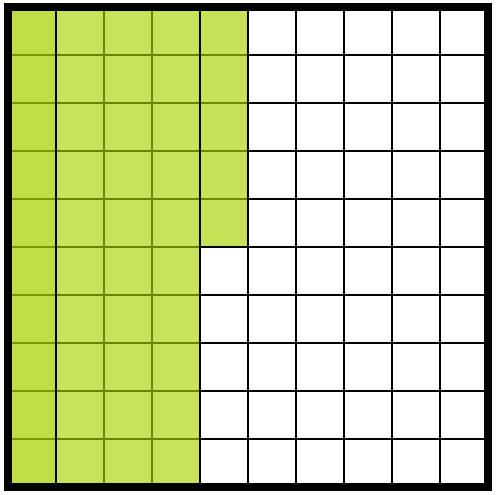
- Two team members record their 3-point shots as 1/10 and 7/20.
  - How did they do as a team?

Story One

- You have a dime (1/10 of a dollar) and seven nickels (7/20 of a dollar).
- In fractional form show how much money you have all together.

How Do We Represent the Money Problem Story One?

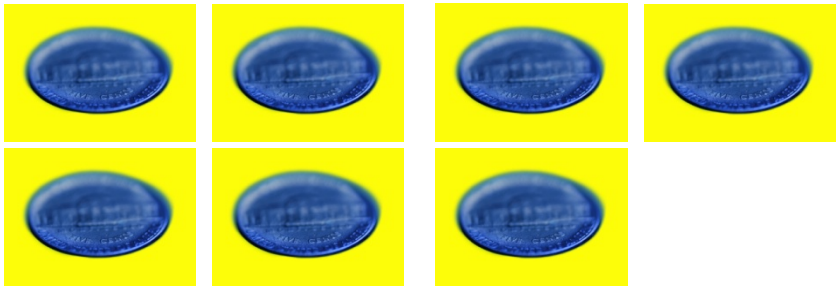
How do we determine this ratio?



_45_ /_100__

1 Dime

7 Nickels

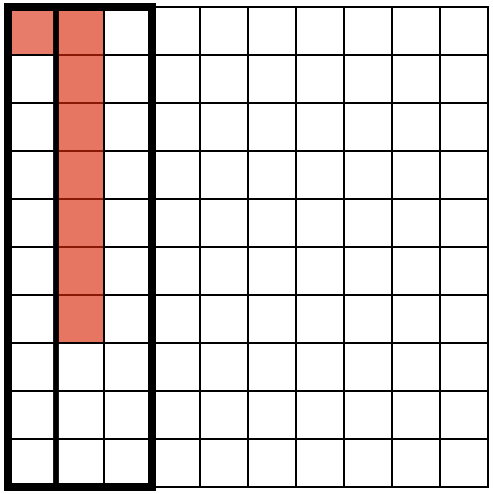


Story Two

- A team has two team members
  - 3-point shots 1/10 and 7/20.
- How did the two of them perform as a team ?

How Did the Team Do Altogether? Story Two

How do we determine this ratio?



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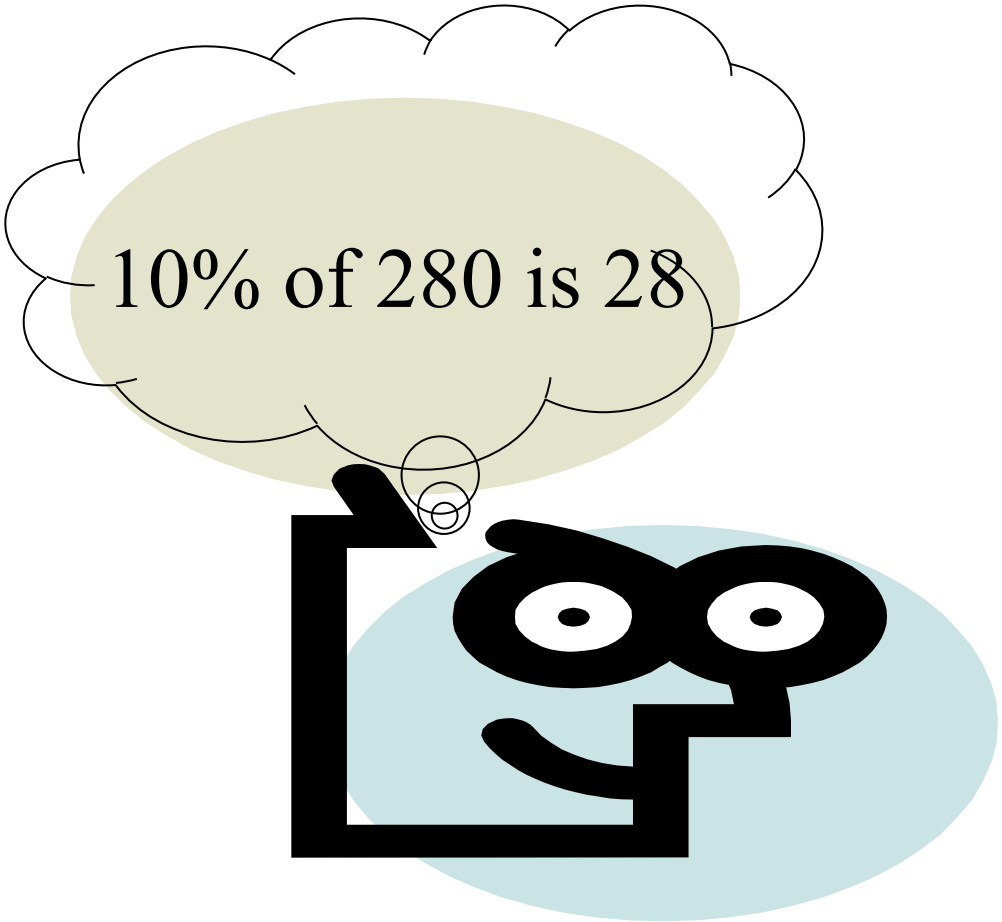
The Unit

- It never goes away…

Week 41: Decimals and Percentages: The Critical Conversions

—what it looks like



Name_____ Utilizing 10% to find other percentages

Find 10% of 140 →

_____

Find 20% of 140→ (10% + 10%)

_____

Find 30% of 140→ (10% + 10% + 10%)

_____

Find 5% of 140→ (1/2 of 10%) _____

Find 15% of 140→ (10% + 5%)

_____

Find 90% of 140→ (100% - 10%) _____

Name_____ Utilizing 10% to find other percentages

Find 10% of 1200 → _____

Find 20% of 1200→ (10% + 10%)

_____

Find 30% of 1200→ (10% + 10% + 10%)

_____

Find 5% of 1200→ (1/2 of 10%)

_____

Find 15% of 1200→ (10% + 5%)

_____

Find 90% of 1200→ (100% - 10%) _____

Name_____ Utilizing 10% to find other percentages

Find 10% of $50 →

_____

Find 20% of $50→ (10% + 10%)

_____

Find 30% of $50→ (10% + 10% + 10%)

_____

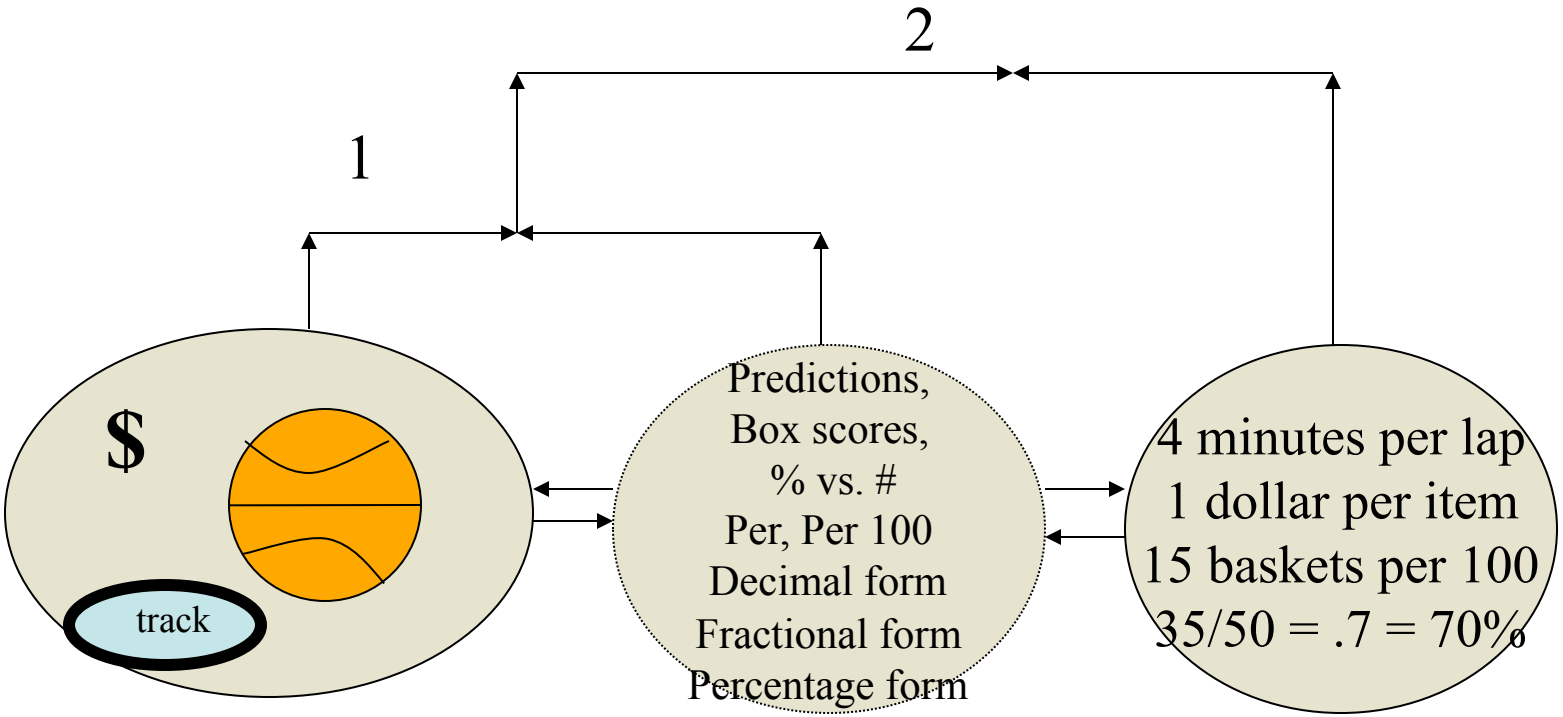
Find 5% of $50 → (1/2 of 10%)

_____

Find 15% of $50→ (10% + 5%)

_____

Rates, Percentages & Predictions



Touchable or visual

Sense-making

Math structures

Recording the info

Faulkner lesson

Journal Prompts

- Week 37—What does “50 miles per hour” mean?
- Week 38—If one person made 2/10 shots and the other made 9/10 shots on the same court, what would you predict would happen if they each took another ten shots? Why?
- Week 39—Explain how you figure out a team’s percentage for trashketball.
- Week 40—If you scored 17/20 on one test, and 23/27 on another test, on which test did you do better? How would you figure that out?
- Week 41–Explain how you would figure 20% of 600.
- Week 42—You went to a restaurant and you wanted to leave the standard 15% tip. Your bill was $25.00 and your friend said to leave $2.00. Is that enough tip? How do you know? What tip would you leave?
- Week 43—Your friend does not understand how to figure percentages. You’re at the store and she wants to know how much a shirt will cost that is 30% off. The shirt costs $20.00 originally. Explain to her how to figure the cost after discount.
- Week 44–If you made a 90 on a test, and there were 20 questions on the test, how many questions did you miss?
- Week 45—Think about the coins that we use (cent, nickel, dime, quarter) and how many of them make up a dollar. What are the

Week 37: What is a Rate?

Concepts:

- A rate is a relationship between two quantifiable things (numbers).
- This relationship allows us to make better predictions
- This relationship can be written in different ways

Materials:

Outdoor Track

Lessons:

Start with one day to settle in. Students will draw pictures of the track and make predictions about how long it will take to walk the track.

Once you've walked the track once, students will be able to make better predictions because they will have a RATE.

Students will then make new predictions and will Walk the track again on Thursday.

If you don't have a track, walk around the school building or up and down the longest hall. Find an easy-to-visualize distance to use. Call it a lap.

Week 37: What is a Rate?—Notes

Begin the warm-ups with kids drawing a track and making predictions about how long it will take to go around the track once. You will discuss the word "per" and develop the concept of rates and prediction.

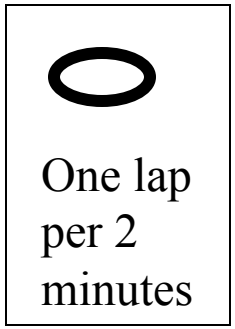
The walking activity should take 10-15 minutes if your track is close to the school. A general human pace would yield about a 4 minute lap (16 minute mile). I'm guessing a middle school student pace (while middle schoolers are, of course, human, they do have their own, unique, set of characteristics) will be about 5-6 minutes per lap. This is a dirge-like 20-24 minute mile.

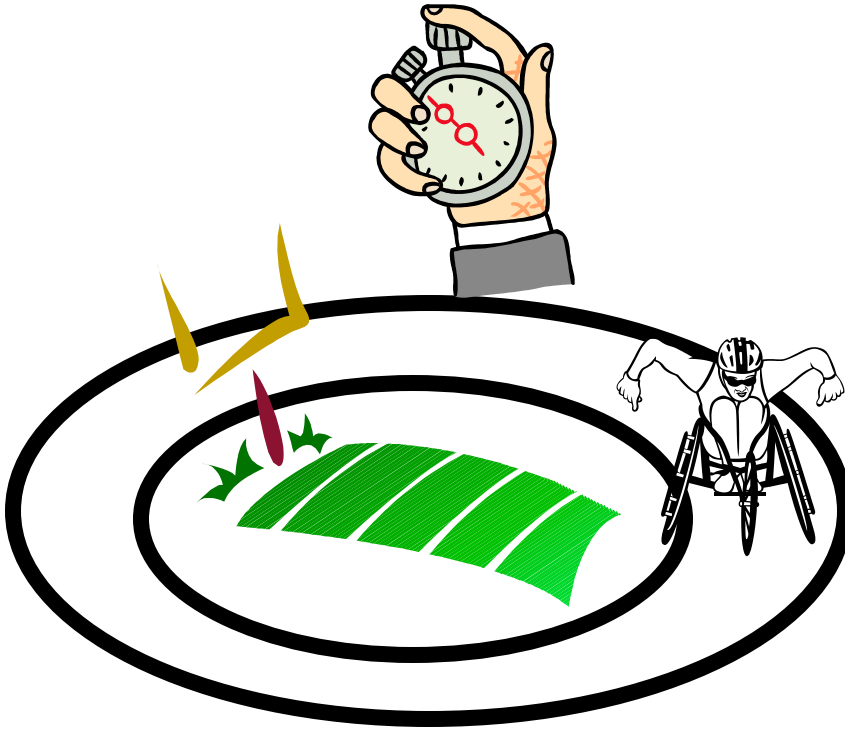
When you return to the track it may take 20 minutes because you are walking around 2 times.

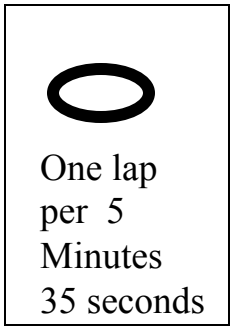
It is important for the activity that you all stay together. For the sake of discussion, you just want one rate going on for now. Later in the nine weeks, kids will have the option to walk at their own pace for an optional project.

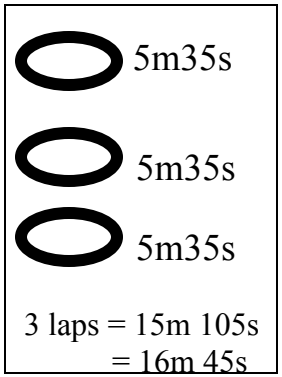
Make the vocabulary connection between laps per minute and miles per hour. Don't work any conversions here, just get them to talk about what miles per hour means, once they understand laps per a number of minutes.

Week 37: What is a Rate?: what it looks like



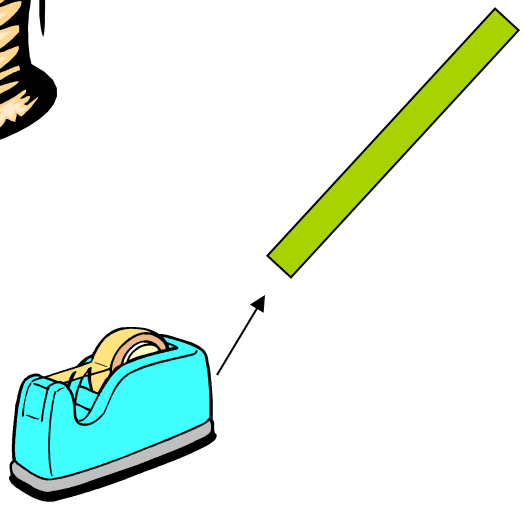


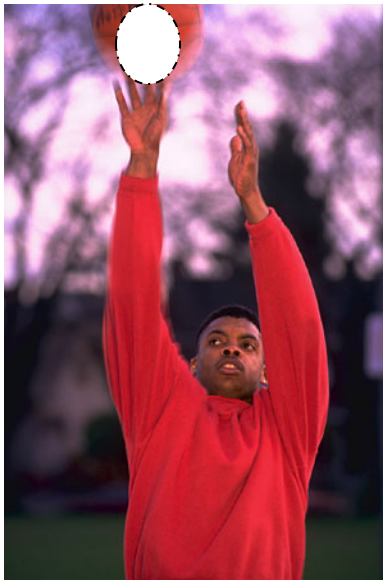




What is our Rate?
How will that rate help us predict our time for 2 or 3 laps?
The form of the time that we will use is minutes and seconds.

Week 38: What is Predictable?: what it looks like





Rates and Prediction Yogis

Court A _**27**__/30

Court B _____/30

Court C _____/30

I predict:

Let's Try It!

- Agreements
  - How far will the foul line be from the basket?
  - Are backboards allowable?
  - What else?
- Demo with two players
  - Representing their stats

Trashketball Procedures

- Team members rotate shots
- After you shoot, you record your result
- Return to end of line for your next shot

- Team Members stay positive with team members: “nice shot” “nice try” “you’ re getting better” “we’ re doing great”

- Stay humble with other teams: “Great percentage, squirrels” “thanks”

Trashketball Stats

Group Name _____

Group Members: 1._____,(2)_____,(3) _____(4) _____

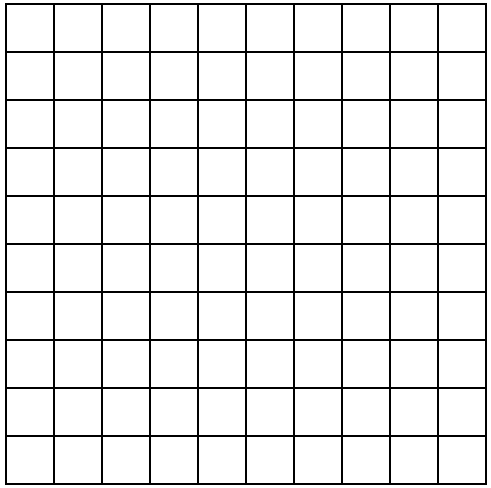
Shots taken: Make or Miss:

1 2 3 4 5 6 7 8 9 10

11 12 13 14 15 16 17 18 19 20

What was our percent?

How do we determine this ratio?



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Trashketball Stats

Group Name _____

Team Members: 1._____,(2)_____,(3)_____,(4)_____

1. **Shots taken: Make or Miss:**

1 2 3 4 5 6 7 8
Total made/Total taken _____5/8 Decimal _____ Percentage_____

2. **Shots taken: Make or Miss:**

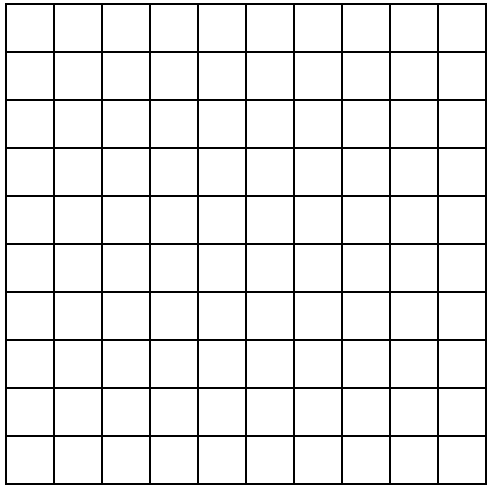
1 2 3 4 5 6 7 8 9 10 11 12 13 14
Total made/Total taken _____7/14 Decimal _____ Percentage_____

3. **Shots taken: Make or Miss:**

1 2 3 4 5 6 7 8 9 10 11
Total made/Total taken _____7/11 Decimal _____ Percentage_____

4. **Shots taken: Make or Miss:**

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Total made/Total taken _____7/16 Decimal _____ Percentage_____



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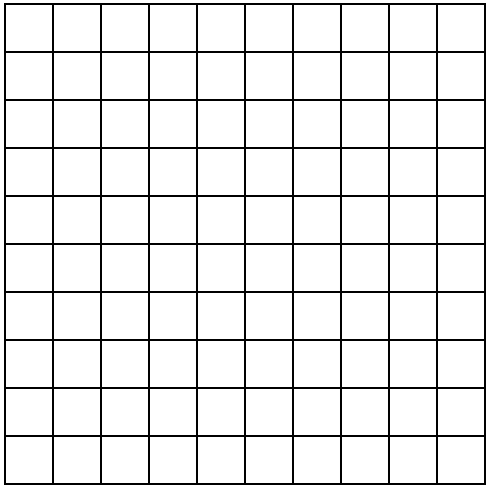
PICTURE IT!



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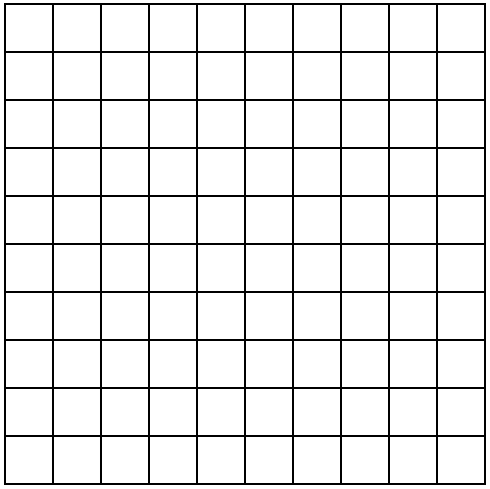

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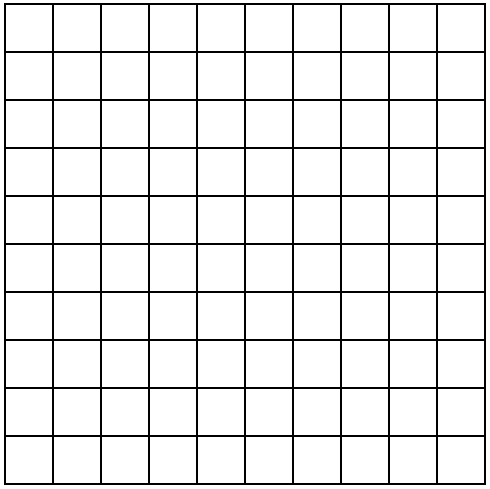

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Bobcats Stats!

A coach can choose any one player to shoot "technical fouls."

If you were the assistant coach of the Bobcats and your team had to shoot technical fouls, who would you suggest to the head coach to shoot the foul shots and why?

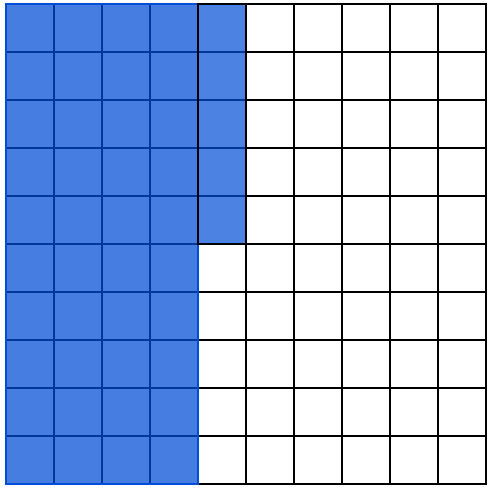
Bobcats Stats

2005–06 Bobcats Regular Statistics

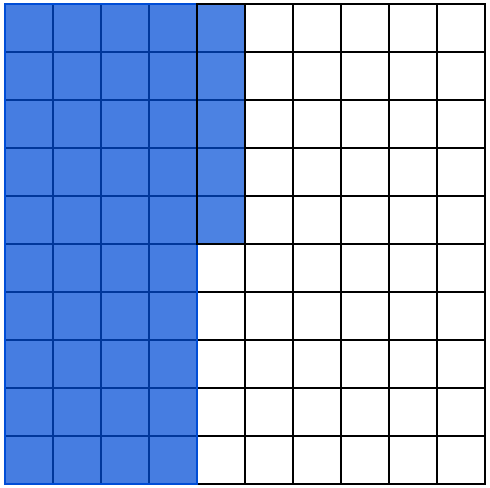
Player	Free Throws Made/Attempted	Free Throw %	3-Points Made/Attempted	3-Point %
Gerald Wallace	164/224	.614	14/50	.280
Emeka Okafor	82/125	.656	0/0	.000
Brevin Knight	252/314	.803	3/13	.231
Primoz Brezec	164/224	.732	0/2	.000
Raymond Felton	161/222	.725	97/271	.358
Jumaine Jones	88/121	.727	115/335	.343
Kareem Rush	55/77	.714	57/164	.348
Melvin Ely	128/192	.667	0/2	.000
Sean May	49/64	.766	0/5	.000
Matt Carroll	138/168	.821	72/185	.389
Bernard Robinson	106/134	.791	2/21	.095
Alan Anderson	33/41	.805	24/58	.414
Jake Voskuhl	41/60	.683	1/3	.333
Lonny Baxter	12/16	.750	0/0	.000
Kevin Burleson	16/17	.941	10/54	.185

“Move it over two”
or
Percentage and Decimal relationship using the Components of Number Sense

45% and .45



out of 100



out of 1

Bobcats Stats!

1) Look at the rates for the Bobcat players for Their free throws (FT%).
Who has the highest rate for FT?

Who made the most free throws all year?

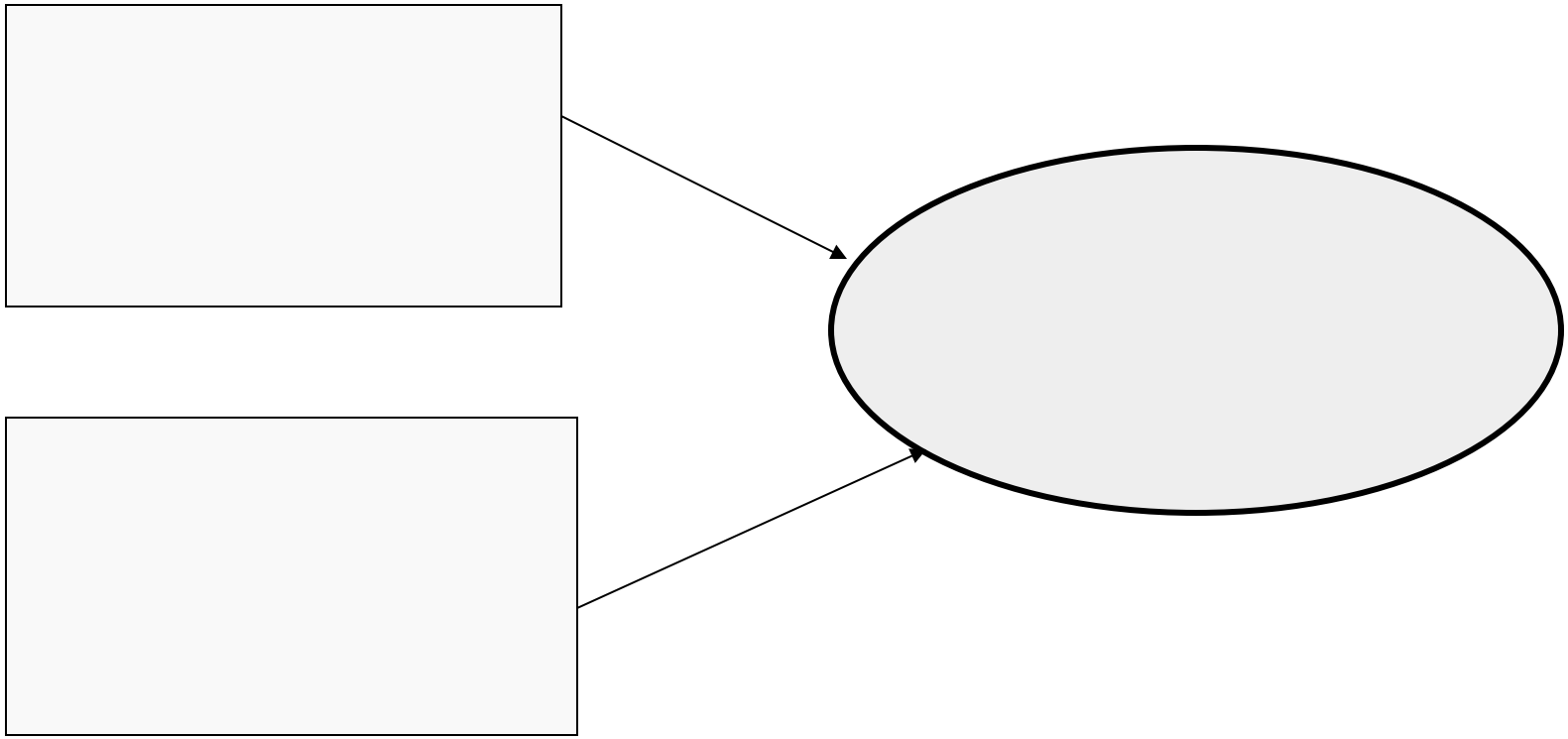
2) A coach can choose any one player to shoot "technical fouls."
If you were the coach of the Bobcats and your team had to shoot technical fouls, who would you choose to shoot the fouls and why?

Bobcats Stats!

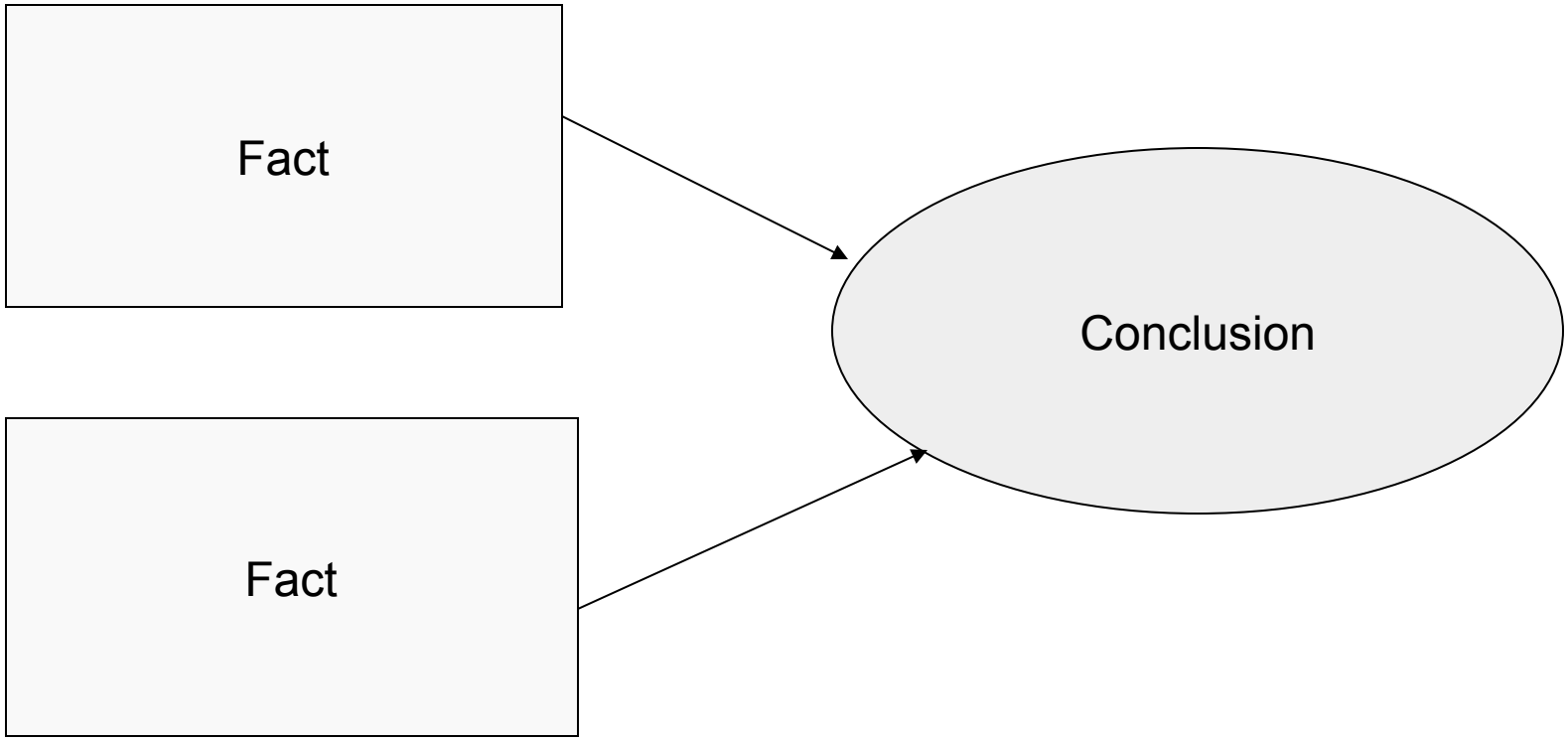
1) Look at the rates for the Bobcat players for Their free throws (FT%).
Who has the highest rate for FT? *Kevin Burleson*

Who made the most free throws all year? *Brevin Knight*

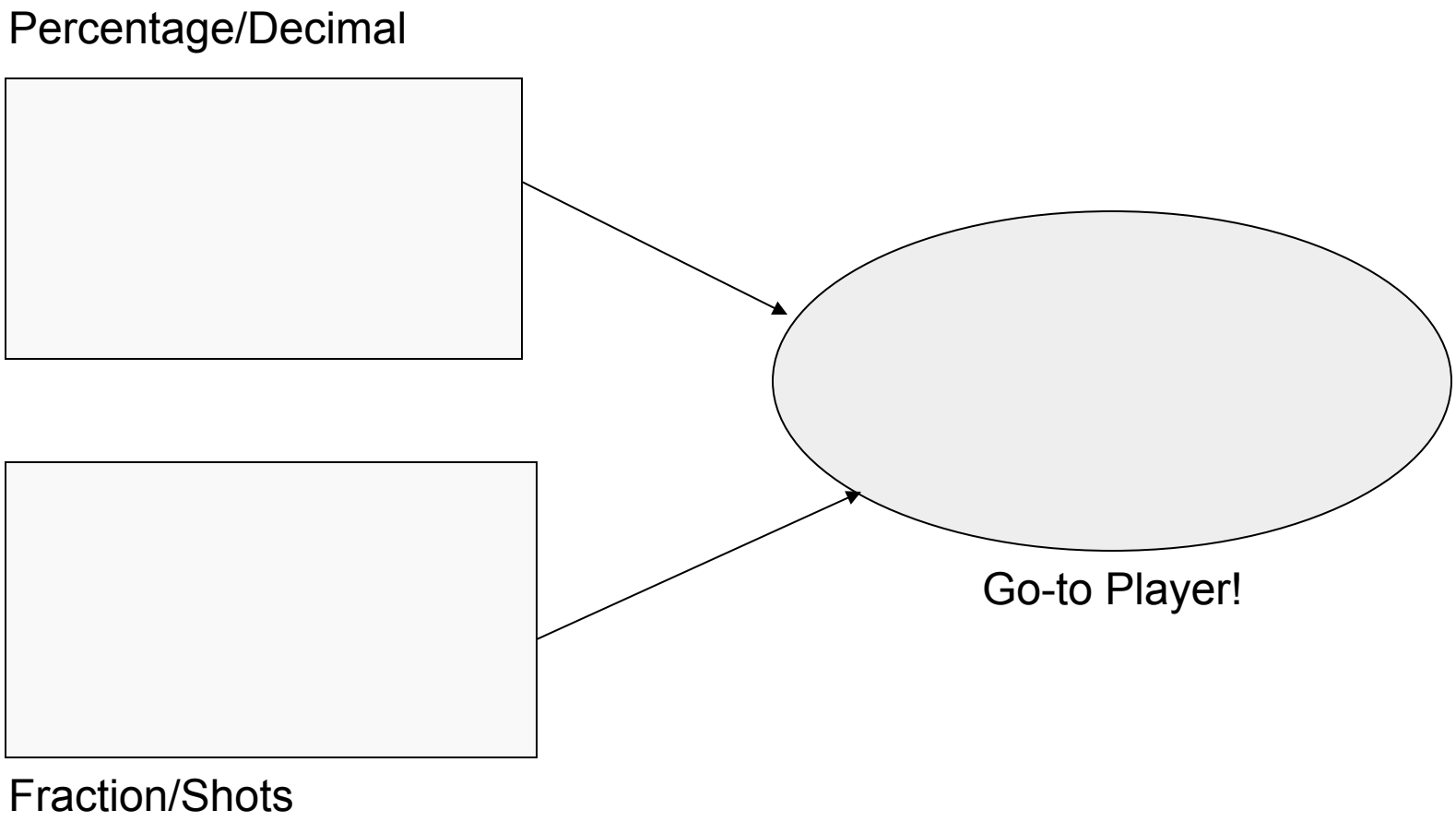
2) A coach can choose any one player to shoot "technical fouls."
If you were the coach of the Bobcats and your team had to shoot technical fouls, who would you choose to shoot the fouls and why?
Kevin Burleson because he has the highest rate and he's only missed one all year; Brevin Knight because he has taken so many shots and, even though he doesn't have the highest rate, he has a lot of experience on the foul line and he has a very good rate.



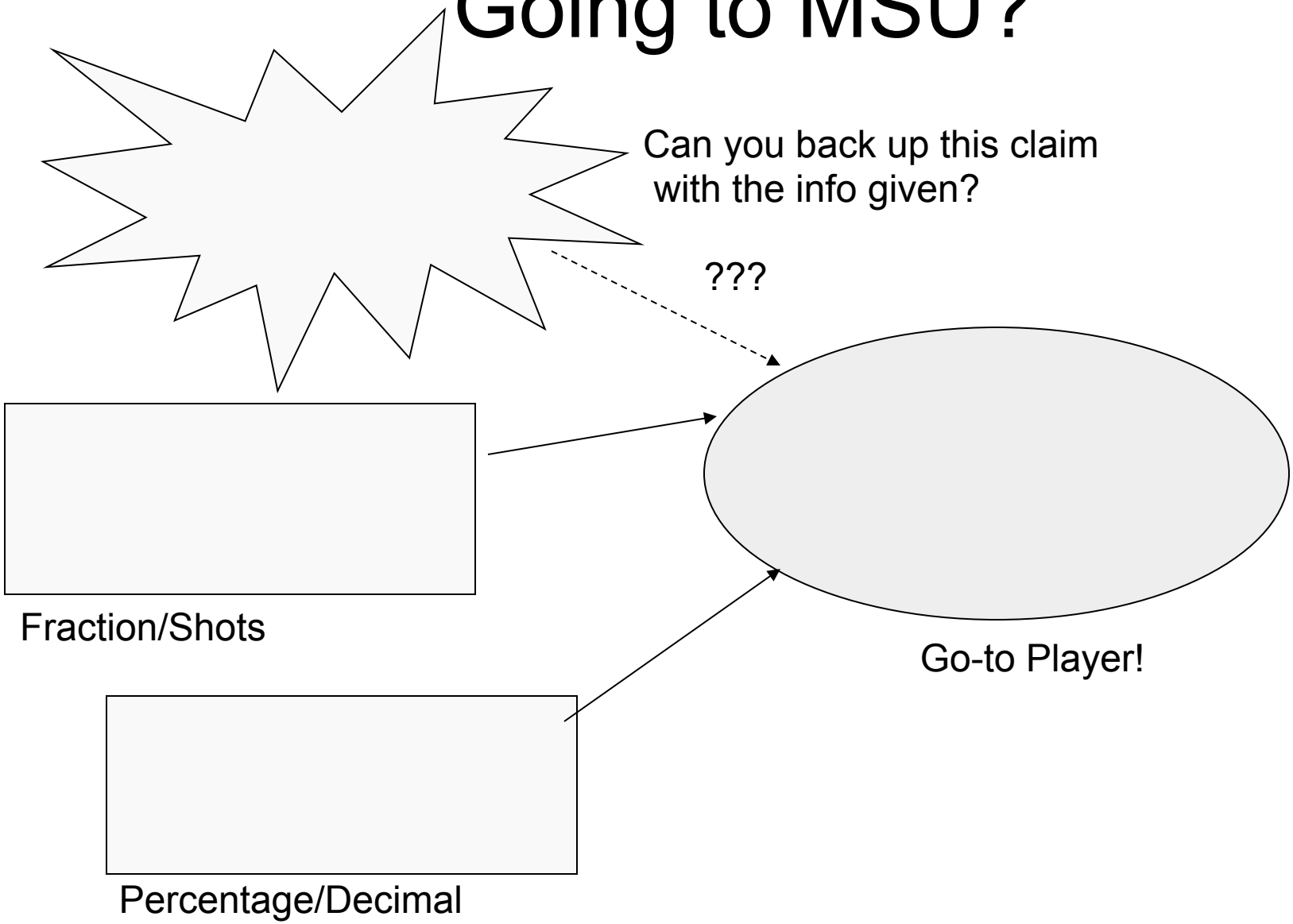
Basic Inference/Conclusion



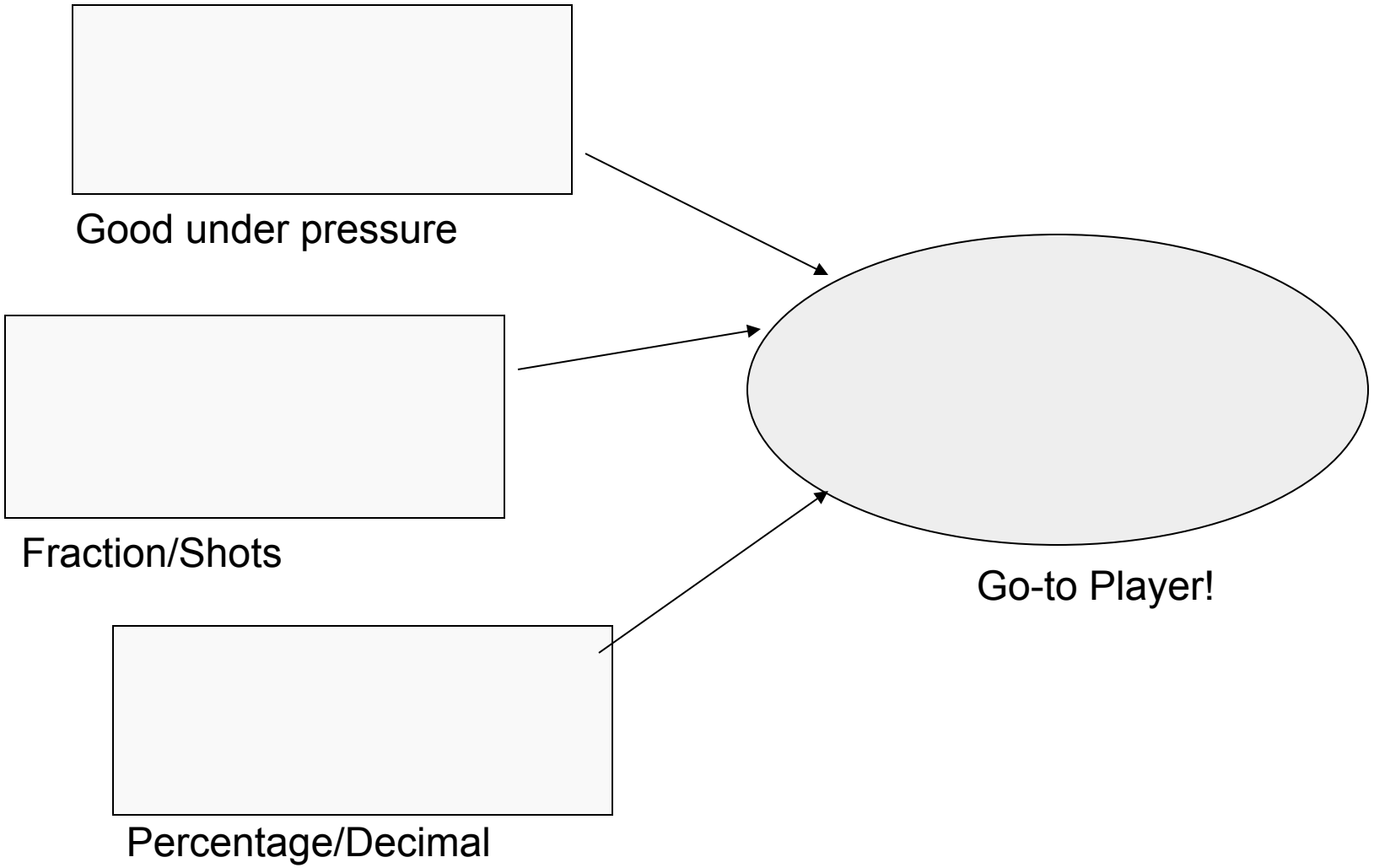
The Coach's Choice: Backing up a decision



The Coach's Choice: Going to MSU?

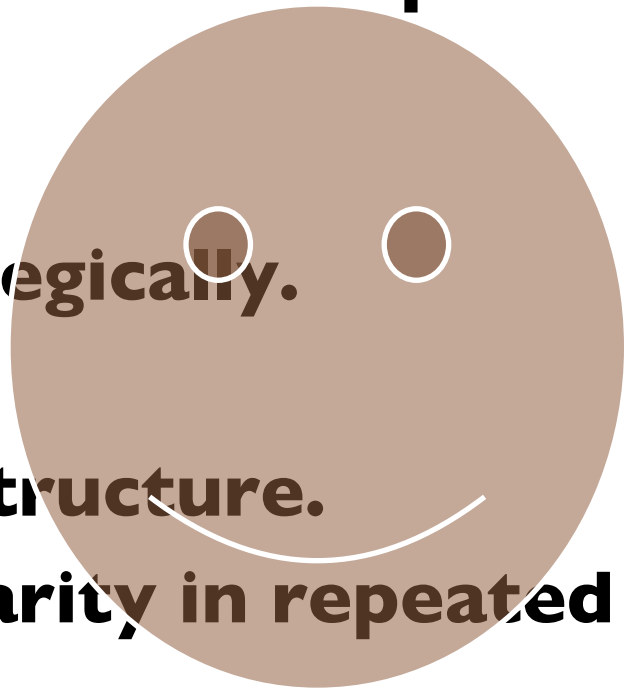


The Coach's Choice: What info might be needed?



Common Core Practice Standards

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**



Classroom

Make a list with your group of language issues you will need to watch with yourself as you discuss math with your students.

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- Calendar and Conference Presentations